Control of constrained spatial three-link flexible manipulators

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Abstract—This study deals with the force and motion control of a spatial three-link articulated manipulator with flexible second and third links. In order to reduce the complexity of the dynamic equations each link is modelled as if unconnected and the joint connections are expressed as constraint equations. Then the joint forces are eliminated and the number of equations is reduced by substituting the acceleration level constraint equations into the dynamic equations. The dynamic equations are partitioned as pseudostatic equilibrium equations and deviations from them. The pseudostatic equilibrium is defined here as a hypothetical state where the velocity and acceleration of the end-effector and the contact forces and/or moments have their desired values while the elastic deformation variables are instantaneously constant. The portion of the control torques for the pseudostatic equilibrium and their portion for the feedback stabilization of the deviations are generated using the measurement signals taken from the end-effector force and moment sensors, the strain gauges, the joint encoders and the end-effector position sensors.

I. INTRODUCTION

Requirements for higher motion speeds and better energy efficiency resulted in a new generation of manipulators with lightweight, flexible links. On the other hand, to achieve high accuracy in operations involving such flexible-link manipulators, more sophisticated control methodologies become necessary. In complex industrial tasks such as grinding, deburring, assembly, and surface finishing, robots usually operate in a constrained environment. Simultaneous motion and force control are required in this type of applications to obtain good performance. The studies involving simultaneous force and motion control of robots with flexible links are limited in number in the current literature. Typical studies are those of Matsuno, Sakawa, and Asano [1], Siciliano and Villani [2], Choi, Lee, and Thompson [3], Hu and Ulsoy [4] and Yim and Singh [5].

Since most of the above studies deal with planar manipulators, the availability of the studies concerning the control of the constrained spatial manipulators with flexible links is even more limited. Among the above studies only Hu and Ulsoy [4] and Yim and Singh [5] applied their methods to spatial manipulators.

Hu and Ulsoy [4] reported a nonlinear controller, using the method developed by Corless and Lettmann [6] for the tracking control of uncertain mechanical systems. Hu and Ulsoy [4] included the feedback of the flexible-body motion to improve the suppression of vibration due to flexibility. A spatial two-link manipulator with flexible forearm was used in their simulations. Their controllers were based on the joint variables. The motion control of a manipulator may be easier in the joint space, however if it has a flexible arm, it becomes preferable to control the end-effector motion directly in the task space to minimize the position error. Yim and Singh [5] considered a control algorithm based on a nonlinear dynamic inversion and a feedback stabilization using linearized equations. They considered a spatial three-link manipulator with flexible third link. However, the difficulty of linearization increases with the degree of freedom of the robot.

Another approach is based on the singular perturbation theory, a typical application of which is presented by Matsuno and Yamamoto [7] for planar constrained flexible manipulators. This approach can be applied when the link stiffness is large enough so that a two-time scale model of the flexible manipulator can be derived. In other words, in the singular perturbation approach, the differential equations of the system are divided into two sets, one for the slow and one for the fast dynamics. If the slow and fast dynamics are not separated naturally by a sufficiently large amount, their separation may be increased artificially by a high-gain feedback control. However, the high gain required for this purpose causes a spillover effect of the ignored higher order modes.

In the present study, a spatial three-link manipulator with flexible second and third links is taken into consideration. In order to reduce the complexity of the dynamic equations each link is modelled as if unconnected and the joint connections are expressed as constraint equations. Then the Lagrange multipliers associated with joint connections are eliminated and the number of equations is reduced by substituting the velocity and acceleration level constraint equations into the dynamic equations.

In an earlier work of Kilicaslan, Ozgoren, and Ider [8], a control method was introduced and tested on a rather simplistic planar two-link manipulator with only one flexible link. In the present work, the performance of the same control method is examined on a realistic spatial three-link manipulator with two flexible links.

In this control method, the dynamic equations of a flexible robot are partitioned as pseudostatic equilibrium equations and deviations from them. The pseudostatic equilibrium is defined here as a hypothetical state where the velocity and acceleration of the end-effector and the contact forces and/or moments have their desired values while the elastic deformations are instantaneously constant. The control torques for the pseudostatic equilibrium are determined in a purely algebraic way and
the feedback stabilization of the deviation equations is achieved using the measurements acquired from the end-effector force and moment sensors, the strain gauges attached to the links, the joint encoders and the end-effector position sensors.

The hypothetical equilibrium state defined above can be interpreted as if it occurs in an equivalent gravitational field that consists of not only $g$ but also the desired acceleration vector. Unlike the singular perturbation methods, in the present method, no assumption is made regarding the magnitudes of the inertia and/or stiffness parameters. Therefore, the present method is applicable for a wider range of systems.

There are two main advantages of the control method used here. One advantage is the easy algebraic calculations of the nominal elastic deformations and the nominal control inputs in the pseudostatic equilibrium. The other advantage is that no linearization of the dynamic equations is required but the linear control techniques can still be used to diminish the deviation of the closed loop pole locations are chosen properly\cite{9,10}. Owing to these advantages, the formulation and implementation of this method are much easier compared to the methods that require linearization and solution of differential equations even for the nominal behavior. This provides convenience especially for high degree of freedom robots having flexible arms. The control method is based particularly on the information about the end-effector position variables, which may be obtained via optical or proximity sensors whenever applicable. Thus, a better tracking quality can be obtained compared to the control methods based on the measurements of the joint variables. Of course, the joint variables are also required in order to compute the current values of the mass matrix, stiffness matrix, etc. They can be obtained easily by means of encoders. The other measurements required by the method are the elastic deformation variables, which can be obtained via strain gauges attached to the links. However, if the working conditions do not allow using optical or proximity sensors, then the end-effector position variables can at least be estimated using the encoder and strain gauge data together with the mathematical model of the system\cite{13}.

\section{Dynamic Modeling}

The spatial manipulator considered in this work is shown in Fig. 1. The motion equations of each link of this manipulator are formulated here separately with respect to the fixed base frame considering its rigid body and elastic degrees of freedom. For sake of mathematical simplicity, this is done as if the links are not connected by the joints. Then, the joint connections are defined through a set of constraint equations.

Link 1 is assumed to be rigid while links 2 and 3 are taken as flexible. Lumped masses $m_A$, $m_B$ and $m_C$ are considered at points $A$, $B$ and $C$, respectively. They represent the actuators at points $A$ and $B$ and the end-effector together with the payload at point $C$.

Referring to Fig. 1, $S = S^{(0)}$ is the fixed frame, $S^{(1)}$, $S^{(2)}$, and $S^{(3)}$ are the frames fixed to the links with the origins at $O$, $A$, and $C$, respectively. $n_i^{(k)}$ is the $k^{th}$ basis vector of $S^{(i)}$. The deformations of each flexible link are described with respect to its own reference frame. The flexible links are assumed to be Euler-Bernoulli beams. Their deformation displacements are considered to be sufficiently small so that they stay in the linear elastic range. The finite element method is used to express their elastic deformations. The flexible links are discretized by two node beam elements as shown also in Fig. 1. The nodal variables are the deformation displacements of the centerlines and the deformation rotations. Since the nodal variables are excessive in number, they are expressed in terms of a smaller number of modal variables ($m^{(k)}$ for link $k$) by means of a reduced modal transformation based on the free vibration analysis of the links.

The first link has no translational degrees of freedom and has no rotational degrees of freedom about $n_1^{(1)}$ and $n_2^{(1)}$ axes. Moreover, the second link can only rotate about the fixed point $A$ and therefore it has no translational degrees of freedom either. These degrees of freedom, that are known to be absent, are not taken into account while deriving the dynamic equations of the links separately. Thus, the first link has a single rotational degree of freedom; the second link has three rotational degrees of freedom accompanied by $m^{(2)}$ modal degrees of freedom; and the third link has three translational and three rotational degrees of freedom accompanied by $m^{(3)}$ modal degrees of freedom.

![Figure 1. Spatial three link manipulator with two flexible arms](image_url)

The motion equations of the links are combined considering the joint connection constraints and the tip point contact constraints as follows\cite{11}

\begin{equation}
M\ddot{y} - \mathbf{B}^T\mathbf{f} = \mathbf{Q} + f^s + f^x + f^c
\end{equation}

where $M$ is the generalized mass matrix of the system; $\mathbf{f}$ is the $c$-dimensional vector of constraint forces at the joints; $\mathbf{B}$ is the Jacobian matrix of the constraints expressed by (3) given ahead; $\mathbf{Q}$ is the Coriolis and centrifugal, external, gravitational, structural stiffness and contact force vectors;
\[ y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ y^{(3)} \end{pmatrix}, \quad y^{(1)} = \beta_1, \quad y^{(2)} = \begin{bmatrix} \bar{\omega}^{(2)} \\ \eta^{(2)} \end{bmatrix}, \quad y^{(3)} = \begin{bmatrix} \zeta^{(3)} \\ \bar{\omega}^{(3)} \\ \eta^{(3)} \end{bmatrix} \]  

where \( y^{(i)} \), \( y^{(2)} \) and \( y^{(3)} \) are the generalized velocity vectors of links 1, 2 and 3, respectively; \( \beta_1 \) is the rotation angle of link 1; \( \bar{\omega}^{(2)} \) and \( \bar{\omega}^{(3)} \) are the angular velocities of links 2 and 3; \( \eta^{(i)} \) are the vectors of modal variables of links 2 and 3; \( \zeta^{(i)} \) is the position vector from the origin of the fixed frame to the origin of the frame of link 3. Equation (1) consists of \( n \) scalar equations but involves \( n+c \) unknowns. Therefore, \( c \) constraint equations must be written. The velocity level constraint equations can be written as described below.

Two scalar equations are written for the revolute joint at point \( A \) to match the angular velocities. For the revolute joint at point \( B \), three scalar equations are written to match the translational velocities and two scalar equations are written to match the angular velocities. All these constraint equations can be written in matrix form as

\[ By = 0 \] (3)

The constraint equations can be written at the acceleration level also in matrix form as

\[ B\ddot{y} = -B\dot{y} \] (4)

Equation (4) consists of \( c \) scalar equations and involves \( n \) unknowns. If (1) and (4) are combined, the complete mathematical description of the system is obtained.

The number of equations of motion derived above by treating each link as if unconnected happens to be rather large and this is not suitable for the control method used in this paper. Therefore, the vector of Lagrange multipliers \( \mu \) associated with the joint connections and the rates other than those of the tip point position variables and the modal variables are eliminated by using (3) and (4).

The tip point position vector \( \zeta = \zeta^{(3)} \) and the vector of modal variables \( \eta = \begin{bmatrix} \eta^{(2)} \\ \eta^{(3)} \end{bmatrix} \) are sufficient to describe the motion of the modelled system completely. The tip point position vector is the variable to be controlled. Therefore, by using the constraint equations, the remaining variables can be expressed in terms of the tip point position vector and the vector of modal variables.

Eliminating \( \beta_1 \), \( \bar{\omega}^{(2)} \), \( \bar{\omega}^{(3)} \), their derivatives, and \( \mu \) from (1), the following equations are obtained [11].

\[ A_{\zeta} \ddot{\zeta} + A_{\dot{\eta}} \dot{\eta} + B_{\zeta} \ddot{\zeta} + B_{\dot{\eta}} \dot{\eta} + D_{\zeta} = E_{\zeta} T + F_{\zeta} \lambda \] (5)

where \( T \) is the vector of actuating inputs, \( K_{\zeta} \) is the structural stiffness matrix and \( \lambda \) is the vector of Lagrange multipliers associated with the tip point contact constraints. In other words, it is the vector of contact forces, which is the second variable to be controlled.

III. CONTROL METHOD

In this paper, the dynamic equations are partitioned as pseudostatic equilibrium equations and deviations from them. Here, the pseudostatic equilibrium is defined as a hypothetical state where the velocity and acceleration of the end-effector and the contact forces and/or moments have their desired values while the modal variables are instantaneously constant. The control torques for the pseudostatic equilibrium and for the stabilization of the deviations are formed in terms of the end-effector motion variables, the modal variables, and the contact force and/or moment components. The joint variables are also used for the online computation of the coefficient matrices in (5) and (6).

The end-effector constraint equations and their derivatives can be written as

\[ \phi(\zeta) = 0 \] (7)

\[ \Phi \dot{\zeta} = 0 \] (8)

where \( \phi \in \mathbb{R}^k \), \( \zeta \in \mathbb{R}^b \) and \( \Phi = \partial \phi / \partial \zeta \). Due to the constraints, the end-effector position can be expressed as \( \zeta = \zeta(s) \), where \( s \) is a vector of \( b-k \) independent variables, which may be designated as "contact surface coordinates". The end-effector velocity components can be related to the rates of the contact surface coordinates as

\[ \dot{\zeta} = \Psi \dot{s} \] (9)

where \( \dot{s} \in \mathbb{R}^{b-k} \) represents the velocity vector tangent to the contact surface and \( \Psi = \partial \zeta / \partial s \). If (9) is substituted into (8), the following equation is obtained.

\[ \dot{\Phi} \Psi \dot{s} = 0 \] (10)

Since \( \dot{s} \) is not identically equal to zero, (10) necessitates that

\[ \dot{\Phi} \Psi = 0 \] (11)

The derivative of (9) gives the acceleration vector as

\[ \ddot{\zeta} = \Psi \ddot{s} + \Psi \dot{s} \] (12)

If (9) and (12) are substituted into (5) and (6), the following equations are obtained:
\[
\begin{align}
R_{\psi\psi}\ddot{s} + R_{\psi\eta}\ddot{\eta} + Y_{\psi\psi}\dot{s} + Y_{\psi\eta}\dot{\eta} + D_{\psi} &= E_{\psi}T + F_{\psi}\lambda, \\
R_{\eta\eta}\ddot{s} + R_{\eta\eta}\ddot{\eta} + Y_{\eta\eta}\dot{s} + Y_{\eta\eta}\dot{\eta} + D_{\eta} &= E_{\eta}T + F_{\eta}\lambda
\end{align}
\]  
(13)

where the coefficient matrices are defined in accordance with these substitutions.

The vectors of contact surface coordinates, the modal variables, the actuating inputs and the Lagrange multipliers (contact forces) can be partitioned as

\[
\begin{align}
s &= s^* + s', \\
\eta &= \eta^* + \eta', \\
\lambda &= \lambda^* + \lambda', \\
T &= T^* + T'
\end{align}
\]  
(15)

where \(s^*\) and \(\lambda^*\) denote the desired values, \(T^*\) and \(\eta^*\) denote the corresponding pseudostatic values and the primed quantities denote the deviations.

The pseudostatic equilibrium is defined as a state such that \(R_{ij}, Y_{ij}, K_{ij}, D, E, F, (i = \psi, \eta \text{ and } j = \psi, \eta)\) are assumed to be frozen at their instantaneous values and \(\eta^*\) is determined as an instantaneously constant elastic deflection vector corresponding to \(s^*, \dot{s}^*, \lambda^*\), and the gravitational acceleration \(g\). Thus, the following equations can be written at the pseudostatic equilibrium:

\[
\begin{align}
\eta &= \eta^*, \\
\dot{\eta} &= 0, \\
\ddot{\eta} &= 0
\end{align}
\]  
(17)

Therefore, at the pseudostatic equilibrium, the dynamic equations given in (13) and (14) take the following forms:

\[
\begin{align}
R_{\psi\psi}\ddot{s}^* + Y_{\psi\psi}\dot{s}^* + D_{\psi} &= E_{\psi}T^* + F_{\psi}\lambda^*, \\
R_{\eta\eta}\ddot{s}^* + Y_{\eta\eta}\dot{s}^* + K_{\eta}\eta^* + D_{\eta} &= E_{\eta}T^* + F_{\eta}\lambda^*
\end{align}
\]  
(18)

\[
\begin{align}
R_{\psi\eta}\ddot{s}^* + Y_{\psi\eta}\dot{s}^* + Y_{\psi\eta}\dot{\eta}^* + D_{\psi} &= E_{\psi}T^* + F_{\psi}\lambda^*, \\
R_{\eta\eta}\ddot{s}^* + Y_{\eta\eta}\dot{s}^* + Y_{\eta\eta}\dot{\eta}^* + K_{\eta}\eta^* + D_{\eta} &= E_{\eta}T^* + F_{\eta}\lambda^*
\end{align}
\]  
(19)

\(T^*\) and \(\eta^*\) can be expressed in terms of \(s^*, \dot{s}^*, \lambda^*\) and \(g\) from (18) and (19) provided that their coefficient matrices combine into a nonsingular augmented matrix. Otherwise, the manipulator will be in an actuation singularity. If (18) is subtracted from (13) and (19) is subtracted from (14), the following deviation equations are obtained:

\[
\begin{align}
R_{\psi\psi}\ddot{s} + R_{\psi\eta}\ddot{\eta} + Y_{\psi\psi}\dot{s} + Y_{\psi\eta}\dot{\eta} + D_{\psi} &= E_{\psi}T + F_{\psi}\lambda, \\
R_{\eta\eta}\ddot{s} + R_{\eta\eta}\ddot{\eta} + Y_{\eta\eta}\dot{s} + Y_{\eta\eta}\dot{\eta} + K_{\eta}\eta + D_{\eta} &= E_{\eta}T + F_{\eta}\lambda
\end{align}
\]  
(20)

\[
\begin{align}
R_{\psi\psi}\ddot{s} + R_{\psi\eta}\ddot{\eta} + Y_{\psi\psi}\dot{s} + Y_{\psi\eta}\dot{\eta} + D_{\psi} &= E_{\psi}T + F_{\psi}\lambda, \\
R_{\eta\eta}\ddot{s} + R_{\eta\eta}\ddot{\eta} + Y_{\eta\eta}\dot{s} + Y_{\eta\eta}\dot{\eta} + K_{\eta}\eta + D_{\eta} &= E_{\eta}T + F_{\eta}\lambda
\end{align}
\]  
(21)

In these equations, the terms in the parentheses can be considered as disturbances. Here, \(T^*\) is to be generated to stabilize the deviations and to keep them as small as possible despite the disturbances. This can be achieved by means of a linear feedback control action as expressed below [8]-[12]:

\[
T' = -S\delta
\]  
(22)

where

\[
\delta = [s^{*T} \dot{s}^{*T} \eta^{T} \eta^{T} \gamma^{T}]^T
\]  
(23)

In this equation, \(\gamma\) is the impulse of the devotional contact force (Lagrange multiplier) vector, i.e.

\[
\gamma = \int_{t_0}^{t} \lambda(t)dt
\]  
(24)

Equations (23) and (24) indicate that integral control action is used for the contact forces while PD control action is used for the motion of the system. As for the gain matrix \(S\), it is determined by using the pole placement method. In other words, \(S\) has to be determined such that the closed-loop poles of the system described by (20) and (21) are placed properly for stability. \(S\) is updated frequently throughout the motion.

On the other hand, in order to guarantee the asymptotic stability of the deviations, the closed-loop system must be rendered slowly varying in the sense of Rosenbrock and Desoer [10], [11]. It has been shown that this stability condition can be satisfied by placing the poles sufficiently away from the imaginary axis [8]-[12].

### IV. NUMERICAL SIMULATIONS

In this section, the force and motion control of the flexible-link spatial manipulator illustrated in Fig. 1 and modelled in Sect. II is investigated by means of a series of simulations. In the simulations, the axial, torsional and shear deformations are assumed to be negligible. For links 2 and 3, the bending deformations are approximated by taking the first two bending modes in the 12 and 13 planes of their link frames. Hence \(m^{(2)} = m^{(3)} = 4\). Fixed-free boundary conditions are used for both links. The links are assumed to have square cross sections and each of them is divided into five finite elements. It is assumed that the lengths of links 1, 2 and 3 are 0.5m, 1.5m and 1.4m, respectively; their masses are 1kg, 1.5kg and 1kg and their lengths of links 1, 2 and 3 are 0.5m, 1.5m and 1.4m, respectively; their masses are 1kg, 1.5kg and 1kg and their densities are 7860kg/m$^3$, 2710kg/m$^3$ and 2710kg/m$^3$. The moduli of elasticity of links 2 and 3 are both 276.1400Pa. The lumped masses at points A, B and C are 1.5kg, 1kg and 2kg, respectively. The natural frequencies associated with the modelled modes of link 2 are 44.0420rad/s and 276.1400rad/s in the 12 plane and 44.0420rad/s and 276.1400rad/s in the 13 plane. Those of link 3 are 42.7293rad/s and 267.9101rad/s in the 12 plane and 42.7293rad/s and 267.9101rad/s in the 13 plane.

In the simulations, the Runge-Kutta fourth-order numerical integration method is used to solve the ordinary differential equations that describe the dynamics of the system. The computer codes are written in MATLAB®.

The tip point is required to track a trajectory on a spherical surface. So, the constraint equation can be written in terms of the tip point coordinates as
\[ \phi(\zeta_1, \zeta_2, \zeta_3) = (\zeta_1 - \zeta_{1c})^2 + (\zeta_2 - \zeta_{2c})^2 + (\zeta_3 - \zeta_{3c})^2 - R^2 = 0 \] (25)

where \( \zeta_{1c}, \zeta_{2c}, \) and \( \zeta_{3c} \) represent the center coordinates of the sphere with respect to the fixed frame and \( R \) is the radius of the sphere. Therefore, the tip point coordinates in the fixed frame and the angular spherical coordinates \( s_1 \) and \( s_2 \) are related as

\[ \zeta_1 = \zeta_{1c} + R \cos(s_2) \sin(s_1) \] (26)

\[ \zeta_2 = \zeta_{2c} + R \sin(s_2) \] (27)

\[ \zeta_3 = \zeta_{3c} - R \cos(s_2) \sin(s_1) \] (28)

Here, \( s_1 \) and \( s_2 \) are known as the azimuth and elevation angles, respectively. In this example, their desired time histories are specified as ninth-order Hermite polynomial splines providing continuous boundary conditions for position, velocity, acceleration, jerk and derivative of jerk. The termination time of the desired motion is specified as 10s. The required variation of the Lagrange multiplier (i.e. the contact force) is specified so that it is composed of a cycloidal rise, an intermediate dwell and a cycloidal return. Its termination time is also specified as 10s.

During the pole placement, \( m \) pole pairs (where \( m \) is the number of elastic modes considered in the controller design) are selected with norms close to the natural frequencies of the system due to the link flexibilities. However, their angles from the imaginary axis are increased so that artificial damping is added to the flexible modes of the system. The remaining poles are chosen freely to obtain a satisfactory performance.

After a few trials, a suitable set of closed loop natural frequency and damping ratio pairs are obtained as follows:

\{10\text{rad/s}, 0.85\}, \{20\text{rad/s}, 0.85\}, \{30\text{rad/s}, 0.85\},
\{43\text{rad/s}, 0.85\}, \{43\text{rad/s}, 0.85\}, \{44\text{rad/s}, 0.85\},
\{44\text{rad/s}, 0.85\}, \{268\text{rad/s}, 0.85\}, \{268\text{rad/s}, 0.85\},
\{276\text{rad/s}, 0.85\}, \{276\text{rad/s}, 0.85\}.

The simulation time step is taken as (1/500)s. The simulation results are presented in Figs. 2-8.
The simulation results indicate that the maximum error magnitudes in the azimuth and elevation angles of the tip point are 0.0493° and 0.0206°, respectively, after the settling time. The results also indicate that the maximum error magnitude in the Lagrange multiplier is 0.588N, again after the settling time. On the other hand, the maximum torques applied to the joints do not exceed 170Nm as seen from Fig. 8.

V. CONCLUSION

In this study, the motion and force control of a realistic three-link spatial manipulator with flexible second and third links is taken into consideration. The derivation complexity of the dynamic equations is reduced by modeling each link as if unconnected and the joint connections are expressed as constraint equations. Then the Lagrange multipliers associated with the joint constraints are eliminated numerically online.

The main advantages of the control method applied here are the easy way of determining the nominal values of the control inputs simply from the algebraic equations that define the so-called pseudostatic equilibrium and the usage of the linear control techniques to stabilize the deviation of the system from its pseudostatic equilibrium without the necessity of linearization. Therefore, it is more convenient to apply compared to the methods that require linearization and solution of differential equations even to determine the nominal values of the control inputs. This convenience becomes especially important as the degree of freedom of the robot and/or the number of modes to be considered for its flexible links increase.

The control torques are formed in terms of the joint variables, the end-effector motion variables, and the modal variables. It is recommended to obtain the current values of the end-effector motion variables by suitable sensors. If such sensors are not available or feasible to use, then the required information can be obtained indirectly by combining the measurements of the joint and modal variables using the mathematical model of the system. It should be noted that the accuracy of the latter approach depends of course on the number of modes included in the mathematical model. As for the modal variables, they are calculated using the displacements which can be obtained through the strain gauge measurements.

The simulation results show that, for reasonable motion and force requirements, it happens to be possible to find suitable closed-loop pole locations in order to obtain a satisfactory performance with tolerably small tracking errors in both the motion and force requirements. The actuating torques can also be maintained at realizable levels. As for the motion and force requirements, they are considered to be reasonable if their variations are smooth and not too fast. Of course, in theory at least, it may be possible to shift the pole locations further as the required variation speeds are increased. However, this cannot be done in practice due to the spillover effect of the ignored higher elastic modes of the flexible links.

REFERENCES