PID Autotuning settings for balanced Servo/Regulation operation

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Abstract—This paper analyzes optimal controller settings for controllers with PID structure. The analysis is conducted from the point of view of the operating mode (either servo or regulation mode) of the control loop and tuning mode of the controller. It is well known that, specially for optimization based settings, the performance of the control loop is defined in terms of the expected operating mode of the control loop. When the control system is not operating in the same operating mode as the controller was tuned, the performance may exhibit very poor results. The performance degradation with respect to the optimal performance is defined and analyzed for settings based on ISE-like optimization criteria. As a consequence, in order to get a minimal overall performance degradation with respect to both operating modes, tradeoff settings are proposed in terms of an autotuning procedure. This autotuning is formulated in terms of the normalized process dead-time.

Index Terms—PID Control, Autotuning, Performance degradation

I. INTRODUCTION

Proportional-Integrative-Derivative (PID) controllers are with no doubt the most extensive option that can be found on industrial control applications. Their success is mainly due to its simple structure and meaning of the corresponding three parameters (therefore making manual tuning possible). This fact makes PID control easier to understand by the control engineer than other most advanced control techniques. In addition, the PID controller provides satisfactory performance in a wide range of practical situations.

During the last years, in fact since the initial work of Ziegler-Nichols [9], much work has been done developing methods to determine the PID controller parameters. Since the manual tuning is a laborious task and requires close attention of the process control engineer, special attention has been devoted to autotuning methods. These methods rely on the application of a special input to the process and by measuring the process response, the PID controller parameters can be determined. Some of the methods employ information about the step response curve such as [9] [3] [4] and [2] for example. A good review of approaches can be found in [1] [5] and [7].

Within the wide range of approaches to autotuning, optimal methods have received special interest. These methods provide, given a simple model process description - such as a first-order plus time delay model -, settings for optimal closed loop response. It is usual to link the tuning method to the expected operation mode for the control system. Therefore there can be found controller settings for optimal set-point or load-disturbance response. This fact allows better performance of the controller when the control system operates on the tuned mode. Obviously there is always the need for a choice: it has to be selected which one of both tuning modes to use.

If the control loop has always to operate on one of the operation modes (as a regulator or as a servo) the choice will be clear. However, when both situations are to occur, it may not be so evident whose are the most appropriate controller settings. What is provided in this paper is the introduction of a Performance Degradation analysis in order to help in this decision.

The analysis presented here concentrates on well known optimal settings such as those presented in [8]. This performance degradation analysis will introduce a quantitative evaluation of the controller settings with respect to the operating mode. As an example, one basic question that could be formulated is: if a set-point setting operating on regulation mode will perform worse than a load-disturbance setting operating on servo mode. Will the answer be plant dependent? To formulate an answer to these questions will help in assist to decide which tuning to apply. These questions, formulated this way, are very generic and in this paper the problem is faced within the well known quadratic performance criteria and corresponding tuning settings as they are presented in [8]. The results presented are based on a numerical approach but encourage a more generic/analytical study.

The paper is organized as follows. First, the optimal PID settings and formulae for both set-point and disturbance are reviewed. It is shown by means of an example how performance can degrade when the controller is not operating according to the tuned mode. Section 3 follows with the formulation of the performance degradation analysis and showing set-point settings provide a better compromise. The results are extended in section 4 looking for an intermediate tuning defined in terms of a tradeoff between both operating modes in such a way that overall performance degradation is minimized. Section 5 generalizes the previous tradeoff approach formulating the corresponding autotuning expressions in terms of the normalized process dead-time. Some examples are presented in section 6 and the paper conclusions are conducted in section 7.
II. OPTIMAL PID SETTINGS

Integral Square Error (ISE) criteria are one of the most well known and most often used criteria [1]. A general formulation of the performance index includes a time weighting factor:

\[ J^n = \int_0^\infty (t^n e(t))^2 dt \] (1)

Criteria (1) with \( n = 0 \) corresponds with the usual ISE criteria and with \( n = 1 \) is known as the ISTE criterion. The optimization of (1) is done subject to the control system criteria and with \( \epsilon \) takes the explicit form of a PID controller.

The plant transfer function, \( P(s) \) is assumed to be represented by a first-order plus dead time (FOPDT) model of the form:

\[ P(s) = \frac{K}{1 + T_s s} e^{-L_s} \] (2)

In addition, a common characterization of the process characteristics is done in terms of the normalized dead-time \( \tau = L/T \) [4]. On the other hand, the ideal PID controller with derivative time filter is considered:

\[ K(s) = K_p \left( 1 + \frac{1}{T_s s} + \frac{T_d s}{1 + (T_d/N)s} \right) \] (3)

The derivative time noise filter constant \( N \) usually takes values within the range 5-10. Without loss of generality here we will consider \( N = 10 \). Assuming the closed-loop system of figure (1) the process output is given by:

\[ y(s) = \frac{K(s) P(s)}{1 + K(s) P(s)} r(s) + \frac{P(s)}{1 + K(s) P(s)} d(s) \] (4)

where the presence of an input load disturbance is considered. The optimal settings presented below correspond to plants with a normalized dead time \( \tau \) in the range 0.1-2.0. Numerical optimization followed by a curve fitting procedure is done for both operating modes. As a result of the curve fitting the controller settings distinguish between \( \tau \in [0.1, 1.0] \) and \( \tau \in [1.1, 2.0] \). See [8].

When determining the settings for optimal set-point response the optimization results are adjusted according to a formulae of the form [6] [8]:

\[ K_p = \frac{a_1}{K} (\tau)^{b_1} \quad T_i = \frac{T}{a_2 + b_2 \tau} \quad T_d = a_3 T (\tau)^{b_3} \] (5)

when determining the optimal operation in regulation mode, the optimization is performed in similar terms as the set-point case. In this case the adjusted formulae that provide the controller settings obey to the following form:

\[ K_p = \frac{a_1}{K} (\tau)^{b_1} \quad T_i = \frac{T}{a_2 + b_2 \tau} \quad T_d = a_3 T (\tau)^{b_3} \] (6)

in both cases, the values of \( a_i \) and \( b_i \) are given in [8] for the ISE, ISTE and ISTE criterion.

III. PERFORMANCE DEGRADATION ANALYSIS

As the performance of the control system is measured in terms of a performance index as (1), formulae (5) and (6) with its respective coefficients, provide the appropriate controller settings once the desired operating mode (servo or regulation) is selected. The possibility of an operation mode different from the selected one, motivates the redefinition of the performance index (1) in terms of notation as:

\[ J^n_{sp}(y) = \frac{1}{T} (t^n e(t, x, y))^2 dt \] (7)

where \( x \) denotes the operating mode of the control system and \( y \) the selected operating mode for tuning. Therefore the tuning mode. Will have \( x \in \{ sp, ld \} \) and \( y \in \{ sp, ld \} \). Obviously, for one specific process it has to be verified that:

\[ J^n_{sp}(sp) \leq J^n_{sp}(ld) \leq J^n_{ld}(sp) \] (8)

Performance will not be optimal for both situations. The Performance Degradation measure is introduced in order to help in the evaluation of the loss of performance with respect to their optimal value. Performance Degradation, \( PD_{om}(tm) \), will be associated to the tuning mode - tm - and tested on the, opposite, operating mode - om -. According to this, the performance degradation of the load-disturbance tuning, \( PD_{sp}(ld) \), will be defined as:

\[ PD_{sp}(ld) = \left| \frac{J^n_{sp}(ld) - J^n_{sp}(sp)}{J^n_{sp}(sp)} \right| \] (9)

conversely, the performance degradation associated to the set-point tuning, \( PD_{sp}(sp) \), will be:

\[ PD_{sp}(sp) = \left| \frac{J^n_{sp}(sp) - J^n_{sp}(ld)}{J^n_{sp}(ld)} \right| \] (10)

In addition, as the controller settings expressed through (5) and (6) have explicit dependence of the process normalized dead-time, \( \tau \), it is worth to bear in mind that the Performance Degradation will also depend on \( \tau \).

Carrying out this performance analysis for the normalized dead-time ranges where controller settings are provided reveals that, as it is shown in figure (2): set-point based settings provide less performance degradation when operating in regulation mode than load-disturbance settings when operating in servo mode.
Note also that Performance Degradation is a decreasing function of the normalized dead-time, taking very high values for processes with small normalized dead-time.

The final decision for the choice of the appropriate tuning mode will depend on the percentage of time that the system is to work as a regulator or as a servo. However, if both situations are likely to occur figure (2) suggests a set-point based tuning is to be preferred. Next section will propose a global approach where some degree of optimality is lost but the performance degradation with respect to both operating modes is considered. Therefore, the overall performance is expected to be enhanced.

IV. SERVO/REGULATION tradeoff TUNING

Tuning approaches presented in section II can be considered extremal situations. The controller settings are obtained by considering, exclusively one mode of operation. This may generate, as it has been shown in the previous section, quite poor performance if the non-considered situation occurs. This fact suggest to analyze if by loosing some degree of optimality is possible to enhance the performance degradation index to be minimized. Obviously the solution will depend on how these factors are defined.

Therefore the resulting controller settings could be considered as an extension of the optimal ones. On this basis we define a controller settings family parameterized in terms of a single parameter $\gamma \in [0, 1]$. The set-point tuning will correspond to a contour constraint for $\gamma = 0$, whereas the load-disturbance tuning for $\gamma = 1$. Figure (3) graphically shows the procedure.

$$\gamma = 0 \quad \gamma \in [0, 1] \quad \gamma = 1$$

$$\text{SP} = [K_p(0), T_i(0), T_d(0)] \quad \text{LD} = [K_p(1), T_i(1), T_d(1)]$$

The controller settings family $[K_p(\gamma), T_i(\gamma), T_d(\gamma)]$ will be generated by observation of formulae (5) and (6). It is seen that the $K_p$ and $T_d$ controller parameters obey to the same expression. Therefore we can think on a parameterized generator $K_p = K_p(\gamma)$ and $T_d = T_d(\gamma)$ and a linear evolution for the integral time constant $T_i$. Therefore:

$$K_p(\gamma) = \frac{a_1(\gamma)}{K} \quad T_i(\gamma) = \gamma T_i^{sp} + (1 - \gamma) T_i^{sp}$$

$$T_d(\gamma) = a_2(\gamma) T_d^{b1}(\gamma)$$

with $\gamma \in [0, 1]$, $T_i^{sp}$ and $T_d^{b1}$ stand for the load-disturbance and set-point setting for $T_i$ respectively, and the $(a_i(\gamma), b_i(\gamma))$ are generated according to:

$$a_i(\gamma) = \gamma a_i^{sp} + (1 - \gamma) a_i^{sp}$$

$$b_i(\gamma) = \gamma b_i^{sp} + (1 - \gamma) b_i^{sp}$$

Now, in order to define a global performance degradation index, the previously defined terms as (10) and (9) need to be extended. Note the performance degradation was associated to the tuned mode. Therefore tested against the opposite operating mode. Now, for every value of $\gamma$ the performance degradation will need to be measured with respect to both operating modes (because the corresponding $\gamma$-tuning does not necessarily corresponds to an operating mode).

- $PD_{sp}(\gamma)$ will represent the performance degradation of the $\gamma$-tuning on servo operating mode.

$$PD_{sp}(\gamma) = \left| \frac{J_{sp}^u(\gamma) - J_{sp}^u(sp)}{J_{sp}^u(sp)} \right|$$

- $PD_{ld}(\gamma)$ will represent the performance degradation of the $\gamma$-tuning on regulation operating mode.

$$PD_{ld}(\gamma) = \left| \frac{J_{ld}^u(\gamma) - J_{ld}^u(ld)}{J_{ld}^u(ld)} \right|$$

From these side performance degradation definitions, the overall performance degradation is introduced and interpreted as a function of $\gamma$: $PD(\gamma)$. There may be different ways to define the $PD(\gamma)$ functional depending on the importance
associated to every operating mode. However, every definition must satisfy the following contour constraints:

$$PD(0) = PD_{id}(sp) \quad PD(1) = PD_{sp}(ld)$$ (16)

The most simple definition would be to give:

$$PD(\gamma) = PD_{id}(\gamma) + PD_{sp}(\gamma)$$ (17)

This expression represents a compromise, or a balance, between both losses of performance.

A. Example

In order to show the performance of the settings presented in section (II) and how performance can degrade when the controller is not operating according to the tuned mode, an example is provided.

Consider the following plant transfer function and corresponding FOPTD approximation:

$$P(s) = \frac{e^{-0.5s}}{(s + 1)^2} \approx \frac{e^{-0.99s}}{1 + 1.658}$$ (18)

The application of the ISE tuning formulae for optimal set-point response provides: $K^p_{sp} = 1.66$, $T^i_{sp} = 1.69$ and $T^d_{sp} = 0.51$, whereas the tuning for optimal load-disturbance provides: $K^p_{ld} = 2.42$, $T^i_{ld} = 1.01$ and $T^d_{ld} = 0.56$.

Figure (4) shows the Performance Degradation analysis for system (18), from where it is found that $\gamma = 0.46$ is the value that minimizes expression (17). With this optimal value of $\gamma$ and the information from FOPTD model, the three parameters of the PID controller are determined using the tuning relations (11), that provides: $K^p = 2.00$, $T^i = 1.38$ and $T^d = 0.53$.

![Fig. 4. Overall (17) and side (14) (15) Performance Degradation terms.](image)

Process output of the system is shown in figure (5) for three tuning modes (set-point, load-disturbance and $\gamma$), when a step change in the set-point occurs at $t=0$ and in the disturbance input at $t=25$.

It is seen that the $\gamma$-tuning gives sensible lower performance than the optimum settings when the system operates in the same way as it was tuned. Also, for each operating mode the performance is better than the purely optimal settings (operated in both modes).

<table>
<thead>
<tr>
<th>Tuning</th>
<th>$PD_{sp}$</th>
<th>$PD_{ld}$</th>
<th>$PD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load-disturbance</td>
<td>0.9341</td>
<td>0.4128</td>
<td>0.9341</td>
</tr>
<tr>
<td>$\gamma=0.46$</td>
<td>0.1277</td>
<td>0.0505</td>
<td>0.1782</td>
</tr>
</tbody>
</table>

% PD reduction: 86.3% 87.8% 56.8%(sp) 80.9%(ld)

Table (I) shows the Performance Degradation values calculated from equations (14) to (17). Also, the last row presents the percentage reduction of PD when the tradeoff tuning is used. All those values confirm the fact that, in global terms (when both operating modes could appear), the $\gamma$-tuning has a better performance than the optimal settings and obviously, it leads to have a smaller PD and an improvement that can be measured as the reduction of that degradation.

As it has been mentioned before, the greatest loss of performance occurs when the load-disturbance tuning operates as a servo mode. It makes that $PD_{sp}$ be the largest component of the global expression of $PD(\gamma)$ and in the opposite side $PD_{ld}$ the smallest one. This thing causes the percentage reduction of $PD$ that can be obtained from the $PD_{ld}$ part to be lower than that of the $PD_{sp}$ part. A balanced
reduction of $PD(\gamma)$ from both performance degradations is possible by applying weighting factors. However we will not go into this issue in this paper. Instead next section concentrates on an automatic way of setting $\gamma$.

It is worth noting that the results shown in figure (4) are practically identical if the family of controllers (11), is generated according to:

$$K_p(\gamma) = \gamma K_p^{sp} + (1 - \gamma)K_p^{ld},$$

$$T_i(\gamma) = \gamma T_i^{sp} + (1 - \gamma)T_i^{ld}$$

and

$$T_d(\gamma) = \gamma T_d^{sp} + (1 - \gamma)T_d^{ld}$$

This is a point that deserves more attention and a more deep analysis has to be done regarding the generation of the controller parameters family. Specially regarding the stability of the resulting closed loop as $\gamma$ varies from 0 to 1.

V. AUTOMATIC tradeoff TUNING SETTINGS

Tuning relations (11) allows to select $\gamma$ on the basis of tradeoff performance for both operating modes. However, it would be desired a way to automatically choose this parameter without the need to run the whole performance degradation analysis.

On this way, using repeated optimizations for equation (17) and different values of the normalized dead time $\tau$, were carried out in order to set the optimal $\gamma$ set. From these optimal values, we can approximate a function (for each $\tau$ range) to determine a general expression for $\gamma$ that gives the best tradeoff tuning. Results are adjusted to formulae of the following form:

$$\gamma(\tau) = a + b(\tau) + c(\tau)^2 \quad \tau \in [0.1, 1.0]$$

$$\gamma(\tau) = a + b(\tau)^c \quad \tau \in [1.1, 2.0]$$

where $a$, $b$ and $c$ are given by table (II) depending of the $\tau$ range that is had.

Figure (6) plots the exact optimal values and the approximate function for both ranges of normalized dead time.

Expressions (20) and (21) for $\gamma$ as $\gamma(\tau)$ allows the set of equations (11), (13) and (12) to provide autotuning ($\gamma$ - autotuning) settings for a balanced servo/regulation operation. This is the main contribution of this communication.

![Figure 6](image.png)

VI. EXAMPLES

This section presents two examples that illustrate, how the implementation of the $\gamma$ - autotuning improves the performance of the system, when the operation mode is different of the tuning mode. Therefore achieving a better tradeoff.

A. Example with $\tau \in [0.1 - 1.0]$

The same situation as in section (IV-A) is developed but using the autotuning settings. From the FOPTD model it is determined that $\tau = 0.6$, therefore equation (20) must be used and $\gamma$ would be equal to 0.4316. Applying relations (11), the PID controller parameters will be: $K_p = 1.98$, $T_i = 1.40$ and $T_d = 0.53$. Figure (7) shows the process output for the three tuning settings and both operating modes (servo and regulation). In table (III) there are the values of Performance Degradation for each tuning. It is found that the results for the $\gamma$-autotuning are practically the same as the showed in table (I) for the previous example (without autotuning), what indicates that approximate function determinate for $\gamma$ is precise enough.

<table>
<thead>
<tr>
<th>$PD_{\gamma}$</th>
<th>$PD_{ld}$</th>
<th>$PD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.341</td>
<td>0.4128</td>
<td>0.4128</td>
</tr>
<tr>
<td>0.1131</td>
<td>0.0654</td>
<td>0.1785</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% PD reduction</th>
<th>$\gamma=0.4316$</th>
</tr>
</thead>
<tbody>
<tr>
<td>87.9%</td>
<td>84.3%</td>
</tr>
<tr>
<td>57.1%(sp) 80.9%(ld)</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE III

PD VALUES FOR THE SYSTEM (18) AND THE PD REDUCTION THAT IS OBTAINED WITH $\gamma$ - autotuning

B. Example with $\tau \in [1.1 - 2.0]$

Now, if we consider a process with a larger $\tau$, as the next one, which is represent by the transfer function and his FOPTD model:
equation (21) to calculate $\gamma$ that would be 0.2469. Using relations (11) to obtain the PID parameters: $K_p = 0.94$, $T_i = 1.08$ and $T_d = 0.56$.

Process output is shown in figure (8), once again for the three tuning settings and both operating modes. It is easy to see how the $\gamma$-autotuning provides a better performance than the one obtained from the purely optimal settings when they operate in both modes (servo and regulation). As it is shown in table (IV), the $\gamma$-autotuning provides a smaller value of Performance Degradation than the other tunings and this causes that the percentage reduction of $PD$ will be substantially reduced.

\[
P(s) = \frac{e^{-1.0s}}{(0.5s + 1)^2} \approx \frac{e^{-1.25s}}{1 + 0.788s}
\]

The $\tau$ value for this system is 1.586, so we need to use equation (21) to calculate $\gamma$ which is 0.2469. Using relations (11) to obtain the PID parameters: $K_p = 0.94$, $T_i = 1.08$ and $T_d = 0.56$.

Process output is shown in figure (8), once again for the three tuning settings and both operating modes. It is easy to see how the $\gamma$-autotuning provides a better performance than the one obtained from the purely optimal settings when they operate in both modes (servo and regulation). As it is shown in table (IV), the $\gamma$-autotuning provides a smaller value of Performance Degradation than the other tunings and this causes that the percentage reduction of $PD$ will be substantially reduced.

![Fig. 7. Performance of the set-point tuning (solid), load-disturbance tuning (dashed) and $\gamma$-autotuning (dot-dashed), for process (18) operating in both servo and regulating modes](image1)

![Fig. 8. Performance of the set-point tuning (solid), load-disturbance tuning (dashed) and $\gamma$-autotuning (dot-dashed), for process (22) operating in both servo and regulating modes](image2)

VII. CONCLUSIONS

An analysis of the performance of a PID based control system with respect to the tuning and operation modes has been presented. When the controller is to perform on an operating mode different from the one it was designed for, a performance degradation is expected. The paper has concentrated on ISE-like performance criteria for the determination of the controller settings. The analysis carried out shows that a set-point tuning offers less performance degradation when operating in regulation mode than a load-disturbance tuning operating as a servo. If both situations are likely to occur a tradeoff will be needed. This is accomplished by means of the introduction of an overall performance degradation index and the corresponding determination of the best tradeoff. Even the results presented and exemplified use the ISE performance criteria the authors have checked, but not shown due to space considerations, that the results extend in the same way for the criteria shown in section (II). As the main contribution of the paper, results are given in the form of autotuning formulae that allows for a direct computation of the associated servo/autoregulation balanced settings.

VIII. ACKNOWLEDGMENTS

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