Control of a bioreactor using feedback linearization

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Abstract—In this paper a feedback linearization control (FLC) strategy was developed and tested for controlling chemical oxygen demand (COD) degradation in an up-flow anaerobic sludge blanket (UASB) reactor used for leachate treatment. The strategy developed allows regulation of the COD concentration at the output stream of the reactor without varying the flow rate, instead we use the temperature as the input control signal. This alternative shows a good perspective for controlling industrial scale reactors with high organic loads and limitations for volume expansion. The control approach is based on a simple model to describe the process and an asymptotic observer to estimate the state variables.

I. INTRODUCTION

Studies carried out by the Global Environment Monitoring System (GEMS) have shown a significant growth in pollution of natural resources, due to the accelerated activity from the development centers. This fact has promoted the creation of environmental rules in order to guarantee a limit to the quantity of toxic matter released in industrial and urban effluents [1]. The wastewater treatment plants (WWTP) have been developed as a solution to mitigate the contamination problem. The municipal landfills produce a complex effluent, composed of a number of household products such as paint, garden pesticides, pharmaceutical products, photographic chemicals, detergents, personal care products, fluorescent tubes, waste oil, heavy metal containing batteries, wood treated with dangerous substances, electronic waste, electrical equipment and chemical residues [2]. This effluent is called leachate, which is formed by the biochemical decomposition of wastes or as result of the water percolation in ground layers or another permeable solid material. The leachate is a product of complex phenomena produced from interactions between filtered water and wastes. As alternative for leachate treatment the anaerobic digestion (AD) offers a suitable solution [3]. The anaerobic digestion is accomplished in the absence of molecular oxygen. The degradation of complex substrates involves a complex network of different types of microorganisms. Full conversion to methane and carbon dioxide results from a biological chain reaction. In this reaction, one type of microorganism generates the substrate consumed by the subsequent organism in the chain [4]. The chain reaction is carried out in four steps: hydrolysis, acidogenesis, acetogenesis and methanogenesis. The AD consolidates itself as a potential alternative, a cause of its advantages associated with the sludge disposal, renewable energy generation, economic costs and environmental benefits [5]. However some obstacles are presented to apply the AD technology, basically for three reasons: the process can become unstable under some circumstances, it has difficulty for an optimal operation and it lacks suitable control algorithms [6]. This last topic has shown an important improvement because of the effort made by the scientific community in this area. The literature reports several control strategies to regulate the chemical oxygen demand (COD) in the output stream of the anaerobic system, using the feed rate as a control action. The purpose of this research is to evaluate the feasibility of a feedback linearization control strategy, considering the temperature as a control action, implemented in an up-flow anaerobic sludge blanket (UASB) reactor for the treatment of a diluted leachate stream produced in the Municipal Landfill La Esmeralda of Manizales-Colombia. The UASB reactor was developed by Dr. Lettinga and his colleagues from Wageningen University, in the late 1970’s [7]. This reactor lets high retention of biomass without any support material, through the formation of sludge granules of 0.5-2 mm diameter with exceptional properties of settling. There is a close correlation between efficiency of a UASB reactor and development of granular sludge. Granulation not only significantly enhances the settleability of biomass leading to effective bacterial retention in the reactor, but also improves physiological conditions making them favorable for bacteria and their interactions, especially syntrophs in the anaerobic system [8].

The paper is organized in the following way, section II is devoted to physical system, section III is related to control technique, section IV presents results and in the section V are the conclusions.

II. PHYSICAL SYSTEM

This study was performed at the Laboratory of Productive Process from the National University of Colombia at Manizales, using an 8.17 l lab scale equipment with a 10.5 ml/min average flow rate. The reactor was made of a 147.5 cm high circular Plexiglas column with 8.4 cm inside diameter. This system counts with a jacket to regulate the temperature of the reactor by means of a 70W electrical resistance. The reactor sketch is illustrated in Fig. 1. In this paper we are focused in phenomenological model of the process (white box model). This kind of models are based on the physiologic state of the bacteria, interaction between phases, ionic equilibrium and the hydraulic behavior of the reactor. The complexity for
modelling this kind of process is even higher compared to other biotechnological process, because of the variety of microorganisms involved (which are not perfectly known) and the substrate to be treated is often a blend of several very different and complicated substrates whose compositions are evolving and are not perfectly known [9]. One of the most critical parameters for modelling of a bio-process is the specific growth rate, which describes the bacterial activity based on kinetic laws. Several expressions for this parameter are found in the literature [10]. Modelling bacterial activity is a hard job, which can be as complex as the modeler wishes. The used model is associated with a goal. When the goal is related with a control purpose, it is necessary taking into account the model to be developed must have a simple structure, which describes the process with acceptable accuracy. This structure may allow to develop control strategies. In this sense, in our model, microorganism colonies are gathered into volatile suspended solids (VSS) and substrates are gathered into chemical oxygen demand (COD).

For modelling reactors, it is required the hydraulic characterization of the system. For the system studied, a tracer test with sodium chloride (NaCl) was carried out. The equations presented in the following correspond to the test with sodium chloride (NaCl) was carried out. The characterization of the system. For the system studied, a tracer test with sodium chloride (NaCl) was carried out. The equations presented in the following correspond to the test with sodium chloride (NaCl) was carried out. The in the literature [11] by means of reactor operation data for a ten months time period. This model showed a 27.53% maximum error percentage, a 5.8% minimum and a 84% average accuracy. This result agrees with the conclusions presented in [12].

\[
\frac{dS}{dt} = D(S_o - S) - Y \Theta^{T-20} \mu X \quad (1)
\]

\[
\frac{dX}{dt} = D(X_o - (1 - \eta)X) + \Theta^{T-20} \mu X \quad (2)
\]

\[
Q_{met} = k_{net} \Theta^{T-20} \mu X \quad (3)
\]

where \( D \) is the dilution rate (days\(^{-1} \)), \( S \) and \( X \) are the concentrations of COD (mg/l) and VSS (mg/l), \( \mu \) is the specific growth rate, \( Y = 3.35\text{mg-COD/mg-VSS} \), and \( k_{net} = 0.0146\text{ml-CH4/mg-l-VSS} \), are the yield coefficients for COD degradation and methane production and \( Q_{met} \) is the methane produced (ml/day). The parameter \( \eta = 0.93 \) (0 \( \leq \eta \leq 1 \)) is associated with the settling efficiency, which allows a high biomass retention. The term \( \Theta^{T-20} \) includes the temperature influence as in [13]. In particular \( \Theta = 1.04 \) is the temperature activity coefficient and \( T \) is the temperature. The subscript \( \text{met} \) indicates influent concentration. The specific growth rate is given by Monod expression as:

\[
\mu = \mu_{max} \frac{S}{K + S} \quad (4)
\]

where \( K = 5522.3 \text{mg-COD/l} \) is the Monod constant and \( \mu_{max} = 1.32\text{day}^{-1} \) is the maximum biomass growth rate. In some works \( X_o \) is neglected, however in this particular case, influent concentration of biomass is taking into account, because landfills can be considered as fixed bed reactors where microorganisms growth is favored and they belong to the leachate. The model found above is analogous to the model presented in [14] for an anaerobic upflow fixed bed reactor.

III. CONTROL SYSTEM

This part shows the asymptotic observer and the control technique used.

A. Asymptotic observer design

One of the main limitations for the monitoring, optimization and control of a biological wastewater system is the unavailability of on-line sensors for critical process variables, which, generally, have a high cost or simply have not been developed. This obstacle has been overcome by means of software sensors (observers) that allow the estimation of variables, using measurements easy to take with software tools. Advantages and drawbacks of these observers are presented in [15]. The asymptotic observer (so called unknown kinetics observer) is designed on the basis of a suitable change of variables via a linear transformation of the state vector [1] and it is necessary to know the yield coefficients of the model. In a previous work [16], a observer was designed and tested, considering the volatile fatty acids. At the end of this paper, we propose the methane measurement as the only variable for the virtual sensor.

In the following, we construct an open loop observer, considering methane measurement. From (1) and (2), we obtain:

\[
\frac{dS}{dt} = D(S_o - \hat{S}) - Y \frac{Q_{met}}{k_{net}} \quad (5)
\]

\[
\frac{d\hat{X}}{dt} = D(X_o - (1 - \eta)\hat{X}) + \frac{Q_{met}}{k_{net}} \quad (6)
\]

Note that the convergence rate of the observer can not be adjusted. Notation \( \hat{\cdot} \) indicates the estimated value for state variables.

![Fig. 1. Reactor sketch.](image-url)
B. Feedback linearization control

Control of AD process has been studied through several approaches such as PID, fuzzy logic, neural networks, feedback linearization control (FLC), adaptive control and others [17], [18]. Most of them use the flow rate as the control action. In this work, we study a control alternative, using the temperature as the manipulated variable, because of its influence in the reaction rate and degradation level of organic matter. This approach allows the COD regulation maintaining a constant flow rate.

The controllability of the state is a very important property of the system, since it allows to reach any state through the input of the system. Before the choice of the control strategy, it is important to know if a control of COD by means of the temperature action is possible. For linear systems the controllability is very well defined. For nonlinear systems we use the Lie brackets for characterizing the controllability property [19].

A nonlinear system, with \( n \) state variables, one input and \( q \) outputs is expressed as:

\[
\dot{x} = f(x) + g(x)u
\]

\[
y(t) = h(x)
\]

where \( x \) is the state vector (\( x \in \mathbb{R}^{n \times 1} \)), \( u \) is the control input (\( u \in \mathbb{R} \)), \( y \) is the output vector (\( y \in \mathbb{R}^{q \times 1} \)) and \( f(x) \), \( g(x) \) and \( h(x) \) are functions of the state with dimensions \( n \times n \), \( n \times 1 \) and \( q \times 1 \) respectively. \( f \) and \( g \) are smooth vector fields on some open set \( X \subseteq \mathbb{R}^n \) containing 0, with \( f(0) = 0 \). In this system the nonlinear controllability matrix is defined by:

\[
W_c(x) = \begin{bmatrix} g(x) & \ldots & ad_{f}g(x) & \ldots & ad_{f}^{n-1}g(x) \end{bmatrix}
\]

(9)

where \( ad_{f}g(x) \) is the Lie bracket of \( f \) and \( g \), defined by:

\[
ad_{f}g(x) = [f, g] = \nabla g f - \nabla f g
\]

(10)

If the matrix \( W_c(x) \) has full range \( n \), the system is reachable, condition equivalent to the definition of the controllability for linear systems. For our system the output is the COD concentration in the exit stream (due to observer estimation) and the control input is represented in terms of \( \Theta^{T-20} \).

Considering the change of variables:

\[
\tilde{S} = S - S_o \quad \text{and} \quad \tilde{X} = (1 - \eta)X - X_o
\]

(11)

equations 1 and 2 are written as:

\[
\frac{d\tilde{S}}{dt} = -D\tilde{S} - Y\Theta^{T-20}\mu_{\text{max}}\frac{\tilde{S} + S_o)(\tilde{X} + X_o)}{(K + S + S_o)(1 - \eta)}
\]

(12)

\[
\frac{d\tilde{X}}{dt} = -D(1 - \eta)\tilde{X} + 6\Theta^{T-20}\mu_{\text{max}}\frac{\tilde{S} + S_o)(\tilde{X} + X_o)}{K + S + S_o}
\]

(13)

We can represent the system in companion form:

\[
f(\tilde{S}, \tilde{X}) = \begin{bmatrix} -D\tilde{S} \\ -D(1 - \eta)\tilde{X} \end{bmatrix}
\]

(14)

\[
g(\tilde{S}, \tilde{X}) = \begin{bmatrix} -Y\mu_{\text{max}}\frac{(\tilde{S} + S_o)(\tilde{X} + X_o)}{(K + S + S_o)(1 - \eta)} \\ \mu_{\text{max}}\frac{(\tilde{S} + S_o)(\tilde{X} + X_o)}{K + S + S_o} \end{bmatrix}
\]

(15)

\[
y = h(\tilde{S}, \tilde{X}) = \begin{bmatrix} \tilde{S} \end{bmatrix}
\]

(16)

Note that \( f(0) = 0 \). The \( W_c(\tilde{S}, \tilde{X}) \) matrix is represented as:

\[
W_c(\tilde{S}, \tilde{X}) = \begin{bmatrix} g(\tilde{S}, \tilde{X}) \mid ad_{f}g(\tilde{S}, \tilde{X}) \end{bmatrix}
\]

(17)

Making the operations for the Lie bracket and analyzing the determinant of \( W_c \), the nonlinear controllability matrix has range = 2 if \( \tilde{S} \neq -S_o \), \( \tilde{X} \neq -X_o \) and \( D \neq 0 \). These conditions are always fulfilled. The physical interpretation is too intuitive: \( D=0 \) implies no feeding, \( \tilde{X} = -X_o \) would imply washout phenomenon, however this state is not possible for our case since the existence of biomass concentration in the input flow. Finally \( \tilde{S} = -S_o \) implies perfect removal, which is not possible in practice. Then the original system always evolves in the first quadrant of the plane \( \tilde{S}-X \). Considering the temperature as the input control signal and the concentration of the transformed system (\( \tilde{S} \)) as the output of the system, and applying the previous condition of controllability, we find that it is possible to control it. In particular we proceed to describe Feedback Control Linearization. The central idea of FLC is to algebraically transform a nonlinear system dynamics into a linear one [20]. The control law must be designed for guaranteeing that error dynamics (deviation of the variable with the set point) is governed by a stable differential equation [10]. The conditions for a feedback linearization are: a) the system is reachable (already proved), and b) the distribution \( \varsigma \) defined as \( \varsigma = g(\tilde{S}, \tilde{X}) \) is involutive. To generate a direct relationship between the output (\( \tilde{S} \)) and the input (\( \Theta^{T-20} \)), the output has to be differentiated one time (equation (12)).

The following control law:

\[
\Theta^{T-20} = \frac{v + D\tilde{S}}{Y\mu_{\text{max}}(\tilde{S} + S_o)(\tilde{X} + X_o)}
\]

(18)

induces a dynamics linear equation between \( \tilde{S} \) and \( v \), where \( v \) is the new input to the closed loop. Deviation between \( \tilde{S} \) and the set point \( \tilde{S}^* \) is given by the error (\( \tilde{e} \)):

\[
\tilde{e} = \tilde{S}^* - \tilde{S}
\]

(19)

The stability condition for error dynamics, is achieved if:

\[
\dot{\tilde{e}} + \lambda \tilde{e} = 0
\]

(20)

where \( \lambda \) is a positive constant. Then

\[
v = \lambda(\tilde{S}^* - \tilde{S})
\]

(21)

Finally, the control law is:

\[
\Theta^{T-20} = \frac{-\lambda(\tilde{S}^* - \tilde{S}) + D\tilde{S}}{Y\mu_{\text{max}}(\tilde{S} + S_o)(\tilde{X} + X_o)}
\]

(22)
Substituting into the equation (12), we have:

\[
\frac{d\tilde{S}}{dt} = \lambda(\tilde{S}^* - \tilde{S})
\]  

(23)

Until now, the control law guarantees the goal for \( \tilde{S} \) regulation. However, we do not know how the biomass behavior would be. Due to the differentiation of \( \tilde{S} \) to obtain a relationship between the output and the input, the system has relative degree one, which produces the so-called internal dynamics [20]. In our case, the internal dynamics is the biomass. The stability of the internal dynamics can be studied from

\[ z_2 = -\eta z_1 - D(1 - \eta)z_2 \]  

(24)

taking into account \( z_1 \) is stable, \( D > 0 \) and \( 0 < \eta < 1 \), then the equation (24) is stable for any operational point and the system is linearizable with a stable internal dynamic. A complete study of the internal dynamic is given in [21].

### IV. Main Results

The control law obtained is influenced by the selection of \( \lambda \) parameter. For studying this relationship, some simulations were carried out, considering as operational conditions: \( D = 2.0 \) day\(^{-1} \), \( S_0 = 2700 \) mg/l COD, \( X_0 = 320 \) mg/l VSS and changing \( S_0 \) to 3500 mg/l COD when \( t = 2 \) days. The reference value was set to \( S^* = 428.8 \) mg/l COD. The open-loop system response is compared with the closed loop system for different values of \( \lambda \) in Fig. 2. For the first two days, all the loops are in the equilibrium state, corresponding with the parameter \( S^* \). When the disturbance in \( S_0 \) is applied, the open loop converges towards another equilibrium, while the closed loops go back to the equilibrium point defined by the set point and the disturbance is efficiently rejected. For increases in \( \lambda \), the settling time and the overshoot decreases. Thus, from a theoretical point of view, making \( \lambda \to \infty \), the closed loop would have better performance. However, it is known that the system has physical limitations and it is not possible to force it more than its inherent conditions would allow. Then the selection of \( \lambda \) is a compromise between the capacity of the actuator and the convergence rate.

Figure 3 relates the state variable \( S \) in equilibrium for fixed values in the temperature of the system and the substrate concentration at the input stream. It is evident that a high removal percentage of organic matter (smaller value in \( S^* \)), implies a high temperature. Note that when \( S_o \) increases, \( S^* \) increases too, however the removal percentage is favored when \( S_o \) is high. For example, with \( X_0 = 320 \) mg/l VSS and \( D = 2.3 \) day\(^{-1} \), when \( S_o = 1500 \) mg/l COD and \( T = 20^\circ C \), \( S^* \) is 368.91 mg/l COD, having a COD removal percentage of 75.41%. On the contrary, for \( S_o = 4500 \) mg/l COD and \( T = 20^\circ C \), \( S^* \) is 584.77 mg/l COD, which represents a COD removal percentage of 87.01%. This characteristic is particular for this kind of systems (high rate). However, due to the possibility to find inhibition conditions, it is important to define a limit for \( S_o \). These conditions are defined by the upper limit of COD concentration at the input stream. Figure 3 give us an idea of the controllability space of the system, since it is linearizable in all state space. However, it is only possible to control it in a region of this space, due to the limitations of the actuator. In the implementation, the upper limit for the temperature was defined as 30C. For this limitation, the minimum value possible for \( S^* \) is 255.75 mg/l COD, with \( S_o = 1500 \) mg/l COD. The saturation phenomenon of the actuator has other physical implications. For example, the convergence rate of the system to reach the operation point \( S^* \) depends on the initial conditions.

The experiment was carried out over a 6 days period. The observer was initialized with the next conditions: \( S(0) = 1422 \) mg/l COD and \( X(0) = 8000 \) mg/l VSS and the set point was fixed at 550 mg/l COD. The \( \lambda \) value was set at 10.

The results from the closed loop system is shown in Fig. 4, for disturbances in \( S_o \) and \( D \). The deviations of the observer are shown in Table I, where \( S \) is compared...
with off-line measurements (which were determined in accordance to APHA AWWA and WPCF [22]).

Note that the control goal is achieved successfully. The settling time is greater than the settling time resulting from the simulations (Figure 2 (D)), which, as it has been said before, is the result of the limitations of the system. The observer has, in the first three data, a significant deviation from the real values of $S$, but in the last points the convergence is evident. It is important to clarify that the performance of the observer is attached with the model accuracy, and in this sense the deviations are in accordance with the model obtained. Thus, it is necessary to improve the model to get a better performance in the observer.

Fig. 4. Response of the closed loop system.

Table I

<table>
<thead>
<tr>
<th>$S_{off-line}$ (mg/l COD)</th>
<th>$S$ (mg/l COD)</th>
<th>Error percentage of the estimation</th>
</tr>
</thead>
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<tr>
<td>880.92</td>
<td>649.39</td>
<td>27.3</td>
</tr>
<tr>
<td>848.85</td>
<td>609.21</td>
<td>28.23</td>
</tr>
<tr>
<td>812.63</td>
<td>550.76</td>
<td>32.22</td>
</tr>
<tr>
<td>623.69</td>
<td>551.19</td>
<td>11.62</td>
</tr>
<tr>
<td>592</td>
<td>550.2</td>
<td>7.1</td>
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<tr>
<td>592</td>
<td>551.2</td>
<td>6.9</td>
</tr>
</tbody>
</table>

V. Conclusions and future work

This study presented the feasibility for COD regulation in an UASB, using the temperature as control action, which has been demonstrated by means of the nonlinear controllability matrix analysis. The PI, fuzzy and feedback linearization strategies were evaluated by means of simulation and reported in other paper [21]. The feedback linearization strategy was superior than others, and its implementation was easy to make. The obtained control law is based on the feedback linearization of the process model, and considering the state estimation through an open loop observer. The control law guarantees the stability of the closed loop system. The results show an alternative to increase the organic matter load in systems with spacial limitations.

The improvement of the model, taking into account four state variables and the development of a closed loop observer is the objective of a future work. The observer will be coupled with multivariable control strategies that consider the variability of the model parameters. In this sense, a first study about application of a model predictive control strategy was developed too. There is an exploration to implement this strategy in the UASB from the Municipal Landfill La Esmeralda, establishing the biogas produced in the reactor as the energy source for heating.

References


