Study and Harmonic Analysis of SVPWM Techniques for VSI-Fed Double-Star Induction Motor Drive

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Abstract—In this paper, a comparison between continuous and discontinuous space vector PWM control of six-phase VSI fed a double-star induction motor drive (DSIM) is presented. The induction machine has two sets of three-phase stator windings spatially shifted by 30 electrical degrees. Each set of three-phase stator windings is fed by a three-phase inverter. The harmonic characteristics of the VSI feeding DSIM are investigated and presented graphically as function of the modulation index with the introduction of a leakage coupling coefficient between the two sets. Implementation on a DSP Controller Board is achieved and experimental results on a 15 kW DSIM prototype machine are carried out.

Keywords—Double star induction motor, six-phase voltage source inverter, space vector PWM, DSP.

I. INTRODUCTION

High power drives employing multiphase machines are required in a lot of applications, such as traction, electric/hybrid vehicles and ship propulsion. In the past decade, multi-level inverter fed electric machine drive systems have emerged as a promising approach in achieving high power ratings with voltage limited devices. The typical structure of such systems is a three-level inverter feeding a three-phase electric machine system [1]. The parallel circuit dual to the multi-level system is, essentially, the base of the concept of the multi-phase inverter. In multi-phase machine drive systems, more than three-phase windings are implemented in the same stator of the electric machine, and one common example of such structure is the Double-Star Induction Motor (DSIM). This motor has two sets of three-phase windings spatially phase shifted by 30 electrical degrees and each set of three-phase stator windings is fed by a three-phase voltage source inverter(VSI), as shown in Fig.1.

Compared with the standard three phase system, the multiphase one brings significant advantages, the main ones are:

- A higher torque density for the same machine volume with reduced torque pulsations: windings with higher phase numbers produce fields with a lower harmonic content and remaining space harmonic fields contribute positively to the torque [2].

- A greater fault tolerance and a higher reliability: because the loss of one phase in multi-phase drive system does not prevent the machine from starting and running, whereas the loss of one phase in a three-phase system results in a single-phase drive which can not start and that will produce a massive pulsating torque if running.

- The possibility to divide the controlled power on more inverter legs [3]. Hence, it will reduce the current stress of each switch and the need for parallel and/or series connection of semiconductor switches may be reduced or removed entirely in power conditioning equipment.

Due to the improvement of fast switching semiconductor power devices, VSI with pulse width modulated (PWM) control arouses a great interest. Control methods that generate the necessary PWM patterns have been discussed extensively in the literature. Two basic concepts may be distinguished: for small and medium power drives, the current controlled PWM has proved to be advantageous. For high power drives employing inverters with low switching frequency, PWM voltage control is more advantageous [4]. Nevertheless, in a DSIM, the two stator windings are mutually coupled and small unbalances in the supplied voltages generate circulating currents [5]. Furthermore, because of the low impedance to harmonic currents there is a high level of circulating currents when a nonsinusoidal voltage source supply is used [6,7], adding losses and demanding larger semiconductor device ratings. Consequently, to minimize these harmonic currents various PWM techniques have been developed.
This paper presents a performance evaluation and harmonic analysis of continuous and discontinuous space vector PWM control of six-phase VSI fed a double-star induction motor drive (DSIM). For purpose of comparison the rms values of the phase current harmonics are presented graphically as function of the modulation index. Also, experimental results carried out on a 15 kW DSIM prototype machine are given.

II. SPACE VECTOR PWM CONTROL OF DOUBLE-STAR INDUCTION MOTOR

With the introduction of multi-phase electrical machines, various modulation techniques have been developed for pulse width modulation (PWM). Among these, the space vector pulse width modulation (SVPWM) provides a number of alternative choices of switching vectors whose time average over one switching period equals a sampled reference voltage vector [8,9].

A. Six Phase VSI Model

The drive system is a six-phase VSI fed DSIM, as shown in Fig.1. A combinatorial analysis of the inverter switch state shows 64 switching modes. So, 64 different voltage vectors can be applied to the machine. By using the (6x6) transformation matrix \([T_s]^{-1}\), we can decompose them into (d-q), (x-y) and (o1-o2) voltages. The (o1-o2) ones are all equal to zero because the neutral of the two windings sets are isolated. So the aim of the PWM is to control during each sampling period four variables simultaneously, by generating maximum (d-q) and minimum (x-y) voltages amplitude. The choice of particular switching modes has to be made in order to satisfy these two conditions. Hence, by choosing the switching modes which permit to select the (d-q) voltage vectors having maximum amplitude, we obtain two complex planes (d-q) and (x-y) divided into 12 sectors and each sector is \(\pi/6\) radian, as shown in Fig.2. Each voltage vector is represented by a decimal number corresponding to the binary number \(K_{c1} K_{c2} K_{b1} K_{b2} K_{a1} K_{a2}\). This binary number gives the state of the upper switches. Only the voltage vectors with maximum amplitude are presented in Fig.2.

\[
[T_s]^{-1} = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & -1/2 & -1/2 & 0 & 0 & 0
0 & \sqrt{3}/2 & 1/2 & -1 & 0 & 0
1 & -1/2 & 1/2 & 0 & 0 & 0
0 & 0 & 0 & 1 & 1 & 1
0 & -\sqrt{3}/2 & -1/2 & 0 & 0 & 0
0 & 1/2 & -1/2 & 0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (1)

B. Proposed SVPWM Techniques Principle

This SVPWM strategy operates in two complex planes (d-q) and (x-y). Since there are four variables to control, four active voltage vectors \(V_s, V_a, V_b\) and \(V_t\) and zero voltage vector need to be chosen during each sampling period, according to the reference voltage vector amplitude and the sector location. Therefore, the reference vector \(v_{\text{ref}}\) is used to locate four adjacent switching state vectors (for example: \(V_{ds}, V_{da}, V_a\) and \(V_{ds} \) in the first sector) and to compute the space vector switching instants \(T_{s54}, T_{s41}, T_9\) and \(T_{s11}\) respectively), during the sampling period \(T_s\). For the remaining time \(T_s = Ts - (T_{s45}+T_{s41}+T_9+T_{s11})\), zero state vectors \(V_o, V_0, V_{oa}\) or \(V_{oa}\) are applied. So SVPWM locally averages, over sampling period \(T_s\), adjacent and zero state vectors to be equal to the reference vector [10].

So that the following equation has a unique and positive solution [1,5]:

\[
\begin{bmatrix}
T_{s45}
T_{s41}
T_9
T_{s11}
\end{bmatrix} = \begin{bmatrix}
V_{ds45}
V_{da41}
V_9
V_{ds11}
\end{bmatrix}^{-1} * \begin{bmatrix}
V_{ds}^* T_s
V_{da}^* T_s
V_9^* T_s
V_{ds11}^* T_s
\end{bmatrix}
\]  \hspace{1cm} (6)

\[T_0 = T_s - (T_{s45} + T_{s41} + T_9 + T_{s11})\]

Figure 2. The inverter voltage vectors on (d-q) and (x-y) planes.
a. Switching Sequences:

In order to minimize (x-y) harmonic currents and maintain the lowest switching frequency, there are different choices to allocate zero voltage vectors \( V_0, V_7, V_{56} \) or \( V_{63} \). The method proposed in this paper is to choose switching sequences in such a way that on the (x-y) plane, two consecutive non-zero vectors are practically opposite in phase. By this way, each change of applied vectors will lead to a succession of increase and decrease in (x-y) currents around zero. The difference between remaining possible switching sequences is due to the selection and placement of zero vectors during the sampling period, as shown in Fig.3. The switching sequences proposed in this paper lead to continuous and discontinuous modulation techniques and, consequently, to different harmonic distortion characteristics. A modulation technique is continuous when on/off switching occurs within every sampling period, for all inverter legs and all sectors. A modulation technique is discontinuous when one (or more) inverter leg stops switching, i.e. the corresponding phase voltage is clamped to the positive or negative dc bus for at least one sector \([11,12]\).

b. Continuous Modulation

For example, when voltage reference vector \( \nu_{idq}^* \) is located in sector-I, a continuous modulation technique (C6φ SVPWM) is obtained with the following sequences:

\[ 7-45-41-56-9-11-7-11-9-56-41-45-7 \]

c. Discontinuous Modulation – sequence A

For the same sector-I, a discontinuous modulation technique (D6φ SVPWM_A) can be obtained with the following sequence:

\[ 7-45-41-9-11-7-11-9-41-45-7 \]

d. Discontinuous Modulation – sequence B1

In the D6φ SVPWM_B1, the zero-voltage vectors are applied at the beginning and at the end of the switching sequence as follows:

\[ 7-45-41-9-11-9-11-9-41-45-7 \]

e. Discontinuous Modulation – sequence B2

In the D6φ SVPWM_B2, the zero-voltage vectors are applied in the middle of the switching sequence as follows:

\[ 45-41-9-11-63-63-9-11-45-41 \]

III. EXPERIMENTAL RESULTS

To confirm the feasibility of the proposed SVPWM techniques on the whole voltage range under V/f and vector control, a set of experimental results are carried out \([13]\). The experimental test bench is composed of a six-phase VSI feeding a 15kW DSIM prototype and the whole control algorithm is tested on a dSPACE DS1104 controller board. This board has a master PowerPC running at 250 MHz and a slave TMS 320F240 DSP. On PowerPC, we implemented the vector control and V/f control. The original 320F240 firmware does not allow the change of PWM compare registers and action registers many times a period.

Moreover, the simple and full PWM are not synchronized \([14]\). So, we reprogrammed the flashed firmware to allow four changes, within a PWM period \( T_s \). Hence, it is possible to implement the continuous and discontinuous PWM. The reloading of compare register (CMPRx, SCMPRx) and action registers (ACTR, SACTR) is done on General Purpose Timer 1 (GPT1) underflow or period match. We have to give the F240 slave DSP the right values before these ticks. Fortunately, the corresponding registers are shadowed. We can then, load them at each GPT2 period match. This second timer has to run 2 times faster than GPT1. It has a count period of 25 µs (half of the triangle pattern) if \( T_s = 200 \mu s \).

We use the "start with GPT1" feature to get GPT2 synchronized with GPT1. The only thing to carefully watch is that CMPRx computed values must not overflow or underflow the timer count limits. It means that there are limitations corresponding to the voltage range that we cannot trespass. This can be solved by choosing a correct placement of zero voltage vectors.

These PWM techniques are successfully tested. The following conclusions can be drawn from these experimental results: In the case of the C6φ SVPWM strategy usually more than two transitions (form low to high or from high to low) occur on the corresponding PWM outputs, which increase the switching frequency of the inverter legs. On the contrary, in the case of the discontinuous PWM strategies: D6φ SVPWM_A, D6φ SVPWM_B1-(B2) at least two PWM outputs remain unchanged during the entire sampling period, which allows a switching frequency minimisation.

All these techniques are tested at the same average switching frequency \( f_{sw} = 5kHz \), and the sampling frequency of each technique is:

\[ - f_s = f_{sw} \text{ for C6φ SVPWM.} \]
- \( f_s = \left(\frac{3}{2}\right) f_{sw} \) for D6 \( \phi \) SVPWM-A.
- \( f_s = 2 f_{sw} \) for D6 \( \phi \) SVPWM-B1-(B2).

However, for the DSP implementation of these strategies, some adaptations are made to ensure successful experiments.

In Fig. 4, the phase current and (d-q) stator current components with constant V/f control are presented. The average switching frequency is set to 5 kHz and the motor is running at 735 rpm with load connected. As expected, the phase current presents a pure sinusoidal shape as well as the (d-q) stator current components for all these PWM techniques, which confirm that these SVPWM techniques allow the control of the (d-q) and (x-y) current components simultaneously.

IV. HARMONIC CURRENT ANALYSIS

The voltage and current waveform quality of the PWM-VSI drives is determined by the switching frequency harmonics. Since they determine the switching frequency copper losses and the torque ripple of a motor load and the line current total harmonic distortion (THD) of a line-connected VSI, the switching frequency harmonic characteristics of a PWM-VSI drive are important in determining the performance. While the copper losses are measured over a fundamental cycle and therefore require a per fundamental cycle (macroscopic) rms ripple current value calculation, the peak and local stresses are properly investigated on a per-carrier cycle (microscopic) base. Therefore, first a microscopic and then a macroscopic investigation is required [12]. Because, the machine model include (d-q) and (x-y) components, so the harmonic current analysis must be made for the (d-q) and (x-y) currents.

1. Normalized harmonic currents and fluxes calculation

The stator voltage equations in the stator coordinate system are expressed as follows:

\[
v_{sdf} = R_s i_{sdf} + \frac{d\phi_f}{dt}
\]

\[
v_{sxy} = R_s i_{sxy} + L_{sxy} \frac{di_{sxy}}{dt}
\]

Where the stator and the rotor flux equations are given by:

\[
\phi_s = L_s i_s + M i_r
\]

\[
\phi_r = L_r i_r + M i_s
\]

The stator flux equation can be rewritten:

\[
\phi_s = \alpha L_s i_s + \frac{M}{L_r} \phi_r
\]

Substituting (4) in (2), the stator voltage equation can be expressed as follows:

\[
v_{sdf} = R_s i_{sdf} + \alpha L_s \frac{di_{sdf}}{dt} + \frac{M}{L_r} \frac{d\phi_r}{dt}
\]

If the relation only between the harmonic voltages and currents is considered, it will be assumed that the reference voltage vector \( \bar{v}_s \) is constant over the sampling period \( T_s \) because the switching frequency \( f_s \) is much higher than the fundamental frequency \( f_1 \) and that the stator and the rotor time constants are much larger than the switching periods, with the resistance drops neglected [15]. Under these assumptions, the voltages and currents can be separated on the harmonic components, which change over \( T_s \) while the fundamental components remain constant over the same period. From eq.(5), the harmonic voltage equation of the stator can be expressed as follows:

\[
\bar{v}_{sdf} = \alpha L_s \frac{di_s}{dt}
\]

\[
\bar{v}_{sxy} = L_{sxy} \frac{di_{sxy}}{dt}
\]

Where \( \bar{v}_s \) is the space vector of the harmonic voltage equal to the difference between the actual voltage vector and the reference vector \( \bar{v}_s \).

Because the harmonic current and harmonic flux are only different in scale, and to eliminate the need of load parameters in eq.(6), the harmonic flux trajectories can be investigated. From eq.(6), it is easy to calculate the harmonic stator flux per-carrier cycle \( \bar{\lambda}_s \) as follows:

\[
\bar{\lambda}_{sdf} = \frac{1}{T_s} \int_{NT_s}^{(N+1)T_s} (V_{sdf} - \bar{v}_{sdf}) dt
\]
\[
\tilde{\lambda}_{sxy} = \frac{1}{T_s} \int_{NT_s}^{(N+1)T_s} (V_{sxy}) \, dt
\]

Where: \( \tilde{\lambda}_{sdq} = a_{dq} \tilde{\lambda}_{sdq} \) and \( \tilde{\lambda}_{sxy} = L_{sxy} \tilde{I}_{sxy} \)

In eq.(7), \( V_s \) is the inverter output voltage vector of the \( k \)th state, and within the carrier cycle it changes according to the selected switching sequence. Since for high \( f_s \) values the \( v' \) term can be assumed constant within a carrier cycle and the \( V_s \) terms are complex number, the above integral can be closed form calculated and the flux trajectories are linear over each state. Therefore, in the SVPWM method only symmetric switching sequences are generated, the integral need only be calculated in the first half of the carrier, and the second half of the trajectory is exact symmetrical to the first \[11,12\].

Moreover, The (x-y) current components are limited by the stator leakage inductance \( L_{sxy} \), which depends on the coil pitch of the stator windings \[5\], and consequently the harmonic characteristics of the VSI feeding DSIM should be investigated with the introduction of the coefficient \( k_{sxy} = a_{xy} / L_{sxy} \), which is necessary to evaluate and compare the performances of the PWM techniques. So, employing (7) and normalizing to \( \tilde{\lambda}_0 = 2\sqrt{3} \tilde{V}_{dcl}T_s / \pi \), the per-carrier cycle rms value of the normalized harmonic flux \( \tilde{\lambda}_{sfrms} \) can be calculated with:

\[
\tilde{\lambda}_{sfrms}^2 (m, \theta) = \frac{1}{T_s} \int_{NT_s}^{(N+1)T_s} (\tilde{\lambda}_{sdq}^2 + k_{sxy}^2 \tilde{\lambda}_{sxy}^2) \, dt
\]

Where: \( k_{sxy} = a_{xy} / L_{sxy} \)

The per fundamental cycle rms value \( \tilde{\lambda}_{sfrms} \) of the harmonic flux determines the waveform quality and harmonic losses. Averaging eq.(8) over a fundamental period results in the global harmonic flux calculation as follows:

\[
\tilde{\lambda}_{sfrms}^2 (m) = \frac{1}{\theta} \int_0^\theta (\tilde{\lambda}_{sdq}^2 (m, \theta) + k_{sxy}^2 \tilde{\lambda}_{sxy}^2 (m, \theta)) \, d\theta
\]

The above integral yields a polynomial function of the modulation index \( m \).

As an example, the per fundamental cycle rms normalized harmonic flux \( \tilde{\lambda}_{sfrms} \) have been calculated for C6 \$ SVPWM, D6 \$ SVPWM-A D6 \$ SVPWM_B1-(B2) PWM techniques, when the reference voltage vector is located in the sector-1. This results in the following \( m \) dependent analytical formulas:

a. Continuous Modulation ‘C6 \$ SVPWM’:

\[
\tilde{\lambda}_{sdqfrms}^2 (m) = \frac{1}{16\pi} (3\pi - 4\sqrt{3}) m^2 + \frac{1}{9\pi^2} \sqrt{2} (53\sqrt{3} - 96) m^3
\]

\[
-\frac{3}{16\pi^3} (27\sqrt{3} + 8\sqrt{3}\pi - 27 - 22\pi) m^4
\]

b. Discontinuous Modulation ‘D6 \$ SVPWM-A’:

\[
\tilde{\lambda}_{sdqfrms}^2 (m) = \frac{1}{27} m^2 + \frac{1}{81\pi^2} \sqrt{2} (59\sqrt{3} - 141) m^3
\]

\[
-\frac{1}{18\pi^3} (8\sqrt{3}\pi + 21\sqrt{3} - 27 - 24\pi) m^4
\]

\[
\tilde{\lambda}_{sxyfrms}^2 (m) = \frac{1}{81\pi^2} (293\sqrt{3} - 507) m^3
\]

(11)

c. Discontinuous Modulation ‘D6 \$ SVPWM_B1-(B2)’:

\[
\tilde{\lambda}_{sdqfrms}^2 (m) = \frac{25}{432} m^2 + \frac{25}{576\pi^2} \sqrt{2} (9\sqrt{3} - 35) m^3
\]

\[
+ \frac{25}{1152\pi^3} (59\sqrt{3} + 16\sqrt{3}\pi - 57 - 12\pi) m^4 - \frac{25}{72\pi^4} \sqrt{2} (161\sqrt{3} - 279) m^5
\]

\[
\tilde{\lambda}_{sxyfrms}^2 (m) = \frac{25}{576\pi^2} \sqrt{2} (175 - 101\sqrt{3}) m^3
\]

(12)

The per fundamental cycle rms curves of the normalized harmonic flux \( \tilde{\lambda}_{sfrms} \) as function of modulation index \( m \) for all the discussed PWM techniques, at the same average switching frequency with different leakage coupling coefficient \( k_{sxy} \), are shown in Fig.5. From these curves, it is clear that the rms value of the harmonic current varies according to the selected switching sequence. In the low modulation index range, C6 \$ SVPWM and D6 \$ SVPWM-A have practically the best performance. As the modulation index increases, the C6 \$ SVPWM performance rapidly degrades while the D6 \$ SVPWM_A is superior in the medium modulation index range. In high modulation index range, D6 \$ SVPWM_B1-(B2) exhibit the best performance. The intersection point of D6 \$ SVPWM-A and D6 \$ SVPWM_B1-(B2) defines the optimal transition point. Therefore, an optimal PWM can be obtained with a transition between these SVPWM strategies to allow rms harmonic current minimization over the whole voltage range. Thus, C6 \$ SVPWM strategy can be applied in a low voltage range, while D6 \$ SVPWM-A can be selected in a medium voltage range and D6 \$ SVPWM_B1-(B2) is advantageous in a high voltage range. We need to underline that discontinuous PWM techniques allow the selection of a higher sampling rate.

V. CONCLUSION

In this paper, the harmonic characteristics of the VSI feeding a Double-Star Induction Motor (DSIM) are investigated and presented graphically as function of the modulation index with the introduction of a leakage coupling coefficient. The switching sequences presented lead to continuous and discontinuous modulation strategies, according to the position of zero voltage vectors during the sampling period.
It is shown that the harmonic current rms value vary according to the selected switching sequence. Also, from this point of view, continuous C6 SVPWM and discontinuous D6 SVPWM_A strategies have an advantage over the others in a low and medium voltage range, while the discontinuous D6 SVPWM_B1-(B2) strategies have an advantage over the others in a high voltage range. In addition, we should remember that with D6 SVPWM_A the carrier frequency can be increased by factor 3/2 for 33% reduction of switching losses, or 2 times increased for 50% reduction of switching losses in the case of D6 SVPWM_B1-(B2). Thus, the combination of these strategies provides the best harmonic current performance over the whole voltage range.

REFERENCES


Figure 5. Per fundamental cycle normalized rms harmonic flux as function of modulation index m, for all the discussed PWM techniques
(a): (d-q) rms harmonic flux. (b): (x-y) rms harmonic flux
(c), (d), (e): The rms harmonic flux with different leakage coupling coefficients (k_{xy} = 1; 5; 10 ), at the same average switching frequency f_{sw}.