Investigating H2 Optimality of Two-Degree of Freedom Control Systems

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Abstract — The control error in a generic two-degree of freedom (TDOF) control system has three major parts: design-, realizability- and modeling-loss. The paper investigates the optimality of the second term in this decomposition and gives conditions when the optimal regulator is an integrating one.

1. INTRODUCTION

Control system optimization is usually based on the error signal or the error transfer function of the closed-loop. The last one is called sensitivity function (SF), so any such optimization procedures is strongly connected to the sensitivity or the robustness of control systems. One widely applied possibility to optimize any norm formulated for the closed-loop is to consider the criterion as a function of the loop-parameters (design, regulator, constraints, etc.) and to solve the strongly nonlinear constrained mathematical programming problem. The existing advanced software tools (MATLAB™, MATHEMATICA™, etc.) are capable to solve such complex tasks, however give relatively little understanding for the influence of the different factors. These methods do not analyze the internal properties of the control error and the different contributing parts of the sensitivity.

2. DECOMPOSITION OF THE CONTROL ERROR IN TDOF SYSTEMS

Assume that the plant to be controlled is factorable as

\[ P = P_r P_ = \frac{B}{A} \frac{B B}{A} \]  \hspace{1cm} (1)

where \( P_r = B_r / A \) means the inverse stable (IS) and \( P_ = B_ \) the inverse unstable (IU) factors, respectively.

In a practical case only the model \( \hat{P} \) of the process is known. Assume that the model \( \hat{P} \), is similarly factorable as the process in (1)

\[ \hat{P} = \hat{P}_r \hat{P}_ = \frac{B}{A} \frac{B B}{A} \]  \hspace{1cm} (2)

where \( \hat{P}_r = B_r / A \) means the IS, \( \hat{P}_ = B_ \) does the IU factors, respectively. Introduce the additive

\[ \Delta = P - \hat{P} \]  \hspace{1cm} (3)

and relative model errors

\[ \ell = \frac{\Delta}{\hat{P}} = \frac{P - \hat{P}}{\hat{P}} \]  \hspace{1cm} (4)

The complementary sensitivity function (CSF) of a one-degree of freedom (ODOF) control system is

\[ T = \frac{C P}{1 + C P} = \frac{C}{1 + T \ell} \]  \hspace{1cm} (5)

where \( \hat{T} \) is the CSF of the model based ODOF system. The SF \( S = (1 - T) \) can be decomposed into additive components according to different principles:

\[ S = \left(1 - R_n\right) + \left(R_n - \hat{T}\right) = S_{des} + S_{real} + S_{id} = \]  \hspace{1cm} (6)

\[ = \left(1 - R_n\right) + \frac{R_n - \hat{T}}{S_{des} S_{perf}} = S_{des} + S_{perf} = \]  \hspace{1cm} (7)

\[ = \left(1 - \hat{T}\right) + S_{id} = \left(1 - \hat{T}\right) + S_{id} = S_{cont} + S_{id} \]

Here \( S_{des} = (1 - R_n) \) is the design, \( S_{real} = \left(R_n - \hat{T}\right) \) is the realizability, \( S_{id} = -\left(T - \hat{T}\right) \) is the modeling (or identification) degradation, respectively. Furthermore \( S_{cont} = \left(1 - \hat{T}\right) \) and \( S_{perf} = (P_n - \hat{T}) \) are the overall control and performance degradations, respectively. The SF depends on the model-based SF \( \left(\hat{T} = 1 - \hat{T}\right) \) as

\[ S = \frac{1}{1 + C P} = \frac{\hat{S}}{1 + T \ell} = \hat{S} + S_{id} \]  \hspace{1cm} (7)

For a two-degree of freedom (TDOF) control system [1] it is reasonable to request the design goals by two stable and usually strictly proper transfer functions \( R_ \) and \( R_n \), that are partly capable to place desired poles in the tracking and the regulatory transfer functions, furthermore they are usually referred as reference signal and output disturbance predictors. They can even be called as reference models, so reasonably \( R_ (\omega = 0) = 1 \) and \( R_n (\omega = 0) = 1 \) are selected.

The term \( S_{id} \) can be further simplified.
\[ S_{id} = S - \hat{S} = \hat{T} - T = \frac{-\hat{T} \hat{S} \ell}{1+T \ell} \bigg|_{\ell \to 0} = -\hat{T} \hat{S} \ell \quad (8) \]

It is easy to see that \( \frac{-\hat{T} \hat{S} \ell}{1+T \ell} \) has its maximum at the cross over frequency \( \omega_c \), which means that the model minimizing \( S_{id} \) is the most accurate around this medium frequency range. (Note that the accuracy of the estimated model at a given frequency is inverse proportional to the weight in the modeling error at that frequency. The realizability and identification degradations can be called as systematic \( (S_{syst}) \) and random \( (S_{rand}) \) components, too.

Assuming that the overall \( CSF \) of a \( TDOF \) control system is \( T_\ell = FT_\ell \), then similar decomposition can be introduced for the tracking error function \( S_i = 1-T_\ell \) as for \( S \) in (6):

\[ S_i = (1-R_i) \left( \frac{1+T_\ell}{1+T \ell} \right) = S_{i_{des}} + S_{i_{real}} + S_{i_{id}} \quad (9) \]

The overall transfer function of the \( TDOF \) system is

\[ T_\ell = \hat{T}_\ell \frac{1+\ell}{1+T \ell} \quad (10) \]

The term \( S_{i_{id}} \) can be further simplified

\[ S_{i_{id}} = \hat{T}_\ell - T_\ell = \frac{-\hat{T}_\ell \hat{S}_\ell \ell}{1+T_\ell \ell} = -\hat{T}_\ell S_\ell \bigg|_{\ell \to 0} = -\hat{T}_\ell \hat{S} \ell \quad (11) \]

In an ideal control system it is required to follow the transients required by \( R_i \) and \( P_n \) (more exactly \( (1-R_n) \)), i.e., the ideal overall transfer characteristics of the \( TDOF \) control system would be

\[ y^o = R_i y_i - (1-R_n) y_n = y_i^o + y_n^o \quad (12) \]

while a practical, realizable control can provide only

\[ y = T_\ell y_i - S w = T_\ell y_i - (1-T) y_n \]

\[ \hat{y} = \hat{T}_\ell y_i - \hat{S} w = \hat{T}_\ell y_i - (1-\hat{T}) y_n \quad (13) \]

for the true \( (y) \) and model-based \( (\hat{y}) \) closed-loop control output signals. Here \( y_i, y \) and \( y_n \) are the reference, process output and disturbance (or output noise) signals, respectively.

Express the deviation between the ideal \( (y^o) \) and the realizable best \( (y) \) closed-loop output signals as

\[ \Delta y = y^o - y = (R_i - \hat{T}_\ell) y_i - (R_n - T) y_n = S_{i_{perf}} y_i - S_{i_{real}} y_n \quad (14) \]

where \( S_{i_{perf}} \) is the performance degradation for tracking and \( S_{i_{real}} = S_{i_{perf}} \) is the performance degradation for the disturbance rejection (or control) behaviors, respectively. Similar equation can be obtained for the deviation between the ideal \( (y^o) \) and the model based \( (\hat{y}) \) closed-loop outputs

\[ \Delta \hat{y} = y^o - \hat{y} = (R_i - \hat{T}_\ell) y_i - (R_n - \hat{T}) y_n = \]

\[ = S_{real}^r y_r - S_{real}^n y_n \quad (15) \]

where \( S_{real}^r \) is the realizability degradation for tracking and \( S_{real}^n \) is the realizability degradation for the disturbance rejection (control) behaviors, respectively.

\[ \Delta y = \Delta \hat{y} = (S_{id}^r y_r - S_{id}^n y_n) \quad (16) \]

It is important to note that the term \( S_{real} \) (and \( S_{real}^r \)) can be made zero for \( IS \) processes only, however, for \( IU \) plants the reachable minimal value of \( S_{real} \) (and \( S_{real}^r \)) always depends on the invariant factors and never becomes zero. In the sequel \( YP \) based control system will be discussed.

### 3. Decomposition in Youla-Parametrized Systems

If the applied regulator design is based on the Youla-parametrization \((YP)\) \([5, 6]\) then the realizable best and the model based regulators are

\[ C = \frac{Q}{1-QP} \quad ; \quad \hat{C} = \frac{Q}{1-QP} \quad (17) \]

Thus the \( CSF \)'s of the true and model-based \( ODOF \) control systems are

\[ T = \frac{\hat{C} P}{1+C P} = \frac{Q \hat{P}(1+\ell)}{1+Q P \ell} \quad ; \quad \hat{T} = \frac{C \hat{P}}{1+C \hat{P}} = Q \hat{P} \quad (18) \]

Only in case of \( YP \) one can also compute the realizable best \( CSF \)

\[ T_s = \frac{C P}{1+C P} = Q P = Q \hat{P}(1+\ell) = \hat{T}(1+\ell) \quad (19) \]

The \( SF \) of the model based and true closed-loops are now

\[ \hat{S} = \frac{1}{1+C \hat{P}} = 1 - Q \hat{P} \quad (20) \]

and

\[ S = \frac{1}{1+C P} = \frac{1-\hat{Q} \hat{P}}{1+Q P \ell} = \frac{\hat{S}}{1+T \ell} \quad (21) \]

The realizable best \( SF \), corresponding to \( T_s \) is

\[ S_s = \frac{1}{1+C P} = 1 - Q P = 1 - \hat{Q} \hat{P}(1+\ell) = \hat{S} - \hat{T} \ell \quad (22) \]
where the identification degradation is

\[ S_{id} = -\frac{Q\dot{P}(1-Q\dot{P})}{1+QP\ell} \]

It is interesting to note that for the realizable best case the decomposition of \( S_{s} = 1 - T_{x} \) results in

\[ S_{s} = 1 - QP = S_{des} + S_{real} + S_{id} = (1-R_{x}) + (R_{x} - QP) - QM\ell = S_{des} + S_{pert} = (25) \]

where

\[ S_{id}^* = -Q\dot{P}(1-Q\dot{P}) \]

This last expression is different from the form (8), because at the optimal point, when \( \dot{P} = \dot{P}_{\text{opt}} \), the \( V \)-parametrized closed-loop virtually opens, therefore the weighting by \( S \) is missing here.

The decomposition of the tracking error function for the \( YP \) is

\[ S_{y}=1-T_{y}=(1-R_{y})+(R_{y} - QP) - (T_{y} - \dot{T}_{y}) = S_{des} + S_{real} + S_{id} = (27) \]

where

\[ S_{id}^* = -Q\dot{P}(1-Q\dot{P}) \]

\[ S_{id} = -\frac{Q\dot{P}(1-Q\dot{P})}{1+QP\ell} \]

\[ \ell = 0 \]

\[ \ell \] is the invariant factor of the \( SF \) can optimally attenuate the disturbance rejection or control behavior of the closed-loop response, respectively.

So the invariant factor \( P_{s} \) can not be eliminated, consequently the ideal design goals \( R_{s} \) and \( R_{d} \) are biased by \( G_{s}P \) and \( G_{d}P \). We can not reach the ideal tracking \( y_{r} = \dot{P}y_{r} \) and regulatory \( y_{r} = \dot{P}y_{r} \) behaviors (see (12)), because of the invariant factor (mainly zeros) in the \( IU \) factor \( P_{s} \). (In a general case the time delay should also be considered here as an invariant factor.) The realizable best transients, corresponding to (13) and (22), are given by \( R_{s}G_{s}P \) and \( (1-R_{d}G_{d}P) \) respectively, where \( G_{s} \) and \( G_{d} \) can optimally attenuate the influence of \( P_{s} \). (Unfortunately \( P_{s} \) does not depend on the control design. This factor is a basic behavior of the process, so it can be considerably changed only via certain technological changes.) The unity gain of \( S_{u} \) ensures integral action in the regulator, which is maintained only if the applied optimization provides \( G_{s}P = 0 \) (or including \( R_{s} \) the condition is \( R_{s}G_{s}P(1) = 1 \)).

The model based version of the \( YP \) regulator \( \dot{C} = C(\dot{P}) \) in the \( GTDOF \) scheme means that \( P \) is substituted by \( \dot{P} \) in equations (29)-(31).

The decomposition of the \( SF \) in the true \( GTDOF \) control system by (23) is

\[ S = S_{des} + S_{real} + S_{id} = (1-R_{x}) + \frac{R_{x}}{1 - R_{x}G_{x}\dot{P}} - \frac{R_{x}G_{x}\dot{P}}{1 + QP\ell} \]

\[ y_{s} = R_{x} \hat{y}_{s} = R_{x}G_{x}P_{y}y \]

\[ y_{r} = R_{x} \dot{y}_{r} = R_{x}G_{x}P_{y}\dot{y} \]

5. OPTIMIZATION OF THE \( SF \) IN \( \mathcal{H}_{2} \) NORM SPACE

Investigate the direct optimization of the \( SF \) in \( \mathcal{H}_{2} \)
norm space first for the GTDOF control system. It is easy to check from (32) that the SF for YP regulator when \( \ell = 0 \) (exact model matching case) is
\[
S = 1 - R_n G_n P_n = S_n \tag{34}
\]
and for the tracking sensitivity
\[
S_t = 1 - R_n G_t P \tag{35}
\]
So the \( \mathcal{H}_2 \) optimality of the SF for a GTDOF system can be ensured by minimizing \( \| y_x (1 - R_s G_s P) \|_2 \) type cost functions, see Fig. 2a. (Here the subscript \( x \) depends on the tracking or the control problem.) Because the regulator cancels the IS factor of the process, this is the reduced or residual form of the optimal scheme, so the task to be solved is
\[
G_x = \arg \left\{ \min_{G_x} \left\{ \| y_x (1 - R_s G_s P) \|_2 \right\} \right\} = \arg \left\{ \min_{G_x} \left\{ \| y_x (1 - R_s G_s P) \|_2 \right\} \right\} \tag{36}
\]
Here an \( y_x = s^{-k} \) form was introduced for the excitation.

![Figure 2. Reduced form of \( \mathcal{H}_2 \) optimality of the SF for the GTDOF control](image)

Note that the above minimization in \( \mathcal{H}_2 \)-norm space is equivalent to the minimum mean square error (MSE) or minimum variance (MV) problem (the classical Wiener paradigm of optimal stochastic systems), if the external excitation \( y_r \) is a white noise sequence, i.e. \( k = 0 \) [4].

Assume a unity gain IS reference model
\[
R_s = \frac{B_s}{A_s} \tag{37}
\]
with coprime \( B_s \) and \( A_s \), thus
\[
J_s = \left\| s^{k} \left( 1 - \frac{B_s}{A_s} G_s B_s \right) \right\|_2 = \left\| s^{k} \right\|_2 \left\| \frac{B_s}{B_s} s^{k} - B_s G_s B_s \right\|_2 \tag{38}
\]
where \( B_s \) contains the unstable zeros of \( B_s \) mirrored on the imaginary axis. Thus \( B_s \) is stable now so \( \| B_s / B_s \|_2 \) is a stable norm-preserving inner function. Taking the usual decomposition
\[
\frac{B_s}{B_s} s^k = \frac{R_s}{B_s} + \frac{K}{s} = \frac{R_s}{B_s} s^k + K \frac{B_s}{s} \tag{39}
\]
where in the right hand side \( R_s / B_s \) is the non-causal and \( K / s^k \) is the causal part consequently (38) becomes
\[
J_s = \left\| \frac{R_s}{B_s} + \frac{K}{s} \right\|_2 \left\| B_s G_s B_s \right\|_2 \tag{40}
\]
Following the orthogonality principle [7] of the classical Wiener design the \( \mathcal{H}_2 \) optimal \( G_x \) can be obtained making the sum of the causal parts zero, i.e.
\[
\frac{C}{s^k} = \frac{B_s G_s B_s}{A_s} = 0 \tag{41}
\]
The \( \mathcal{H}_2 \) optimal embedded filter \( G_x \) is obtained as
\[
G_x = \frac{A_s}{B_s} \frac{K}{B_s} \frac{B_s}{A_s} \tag{42}
\]
\( \mathcal{R} \) and \( \mathcal{K} \) can be obtained from the special Diophantine equation (39)
\[
B_s = \mathcal{R} s^k + K B_s \tag{43}
\]
It is interesting to observe that using the obtained \( \mathcal{H}_2 \) optimal \( G_x \) and the original form \( y_x (1 - R_s G_s P) \) changes to \( \left\| y_x (1 - G'_s P) \right\|_2 \), where
\[
G'_s = \mathcal{K} / B_s \tag{44}
\]
because the optimization cancels all stable factors. This optimal embedded filter corresponds to the scheme shown in Fig. 2b, which is the classical Nehari problem [2], [7]. The minimum of the cost function is
\[
J_s = \left\| R_s / B_s \right\|_2 \tag{45}
\]
The \( \mathcal{H}_2 \) optimal regulator using (29) and (42) is
\[
C_s = \frac{\mathcal{K} A}{B_s \left( B_s - \mathcal{K} P \right)} \tag{46}
\]
Investigating the product
\[
G_s B_s = \frac{A_s}{B_s} \frac{K B_s}{B_s} = \frac{A_s}{B_s} \left( 1 - K B_s s^k \right) \tag{47}
\]
where (43) was used, it is easy to see that \( G_s B_s \|_{s=0} = 1 \), providing integrating regulator can be obtained if, and only if \( k \geq 1 \). This is an important result; the original formulation of the \( \mathcal{H}_2 \) optimality of \( \| s \|_2 \) using the operator norm of the SF can not provide an integrating regulator. This is why the excitation \( y_x (s) = s^{-k} \) was
introduced and the induced norm \( \|x\|_2 \) was used in the optimization.

This result is not surprising, because the sensitivity function is usually a high-pass filter, however, the applicability of the Parseval theorem to calculate the \( H_2 \) norm directly for an error transfer function requires a minimum pole access one. This requirement is ensured by \( k \geq 1 \).

**Example 1.**

Assume a first order reference model \( R_s = 1/(1+sT_w) \) and be \( B_\infty = 1+sT \), so \( B^- = 1+sT \). The solution of the Diophantine equation (43) for \( k = 0 \) gives \( R = 2 \) and \( K = -1 \). The optimal filter is now

\[
G_x = \frac{1+sT_w}{1+sT}
\]

and the \( H_2 \) optimal regulator is

\[
C_x = \frac{A(-1)}{B_x[(1+sT)-(1)(1-sT)]} = \frac{A}{2B_x}
\]

which is not an integrating one.

Solving the problem for \( k = 1 \) gives \( R = 2T \) and \( K = 1 \). The optimal filter is now

\[
G_x = \frac{1+sT_w}{1+sT}
\]

and the \( H_2 \) optimal regulator is

\[
C_x = \frac{A(1)}{B_x[(1+sT)-(1)(1-sT)]} = \frac{A}{2B_x sT}
\]

which is an integrating one.

An important conclusion that the optimization of the \( SF \) in \( H_2 \) norm space does not depend on the reference model \( R_s \) only on the invariant process factor \( B_\infty \) and \( k \). This observation induced the need to investigate criteria, what depends on the design goal, too.

6. MINIMIZATION OF THE REALIZABILITY LOSS

The optimization of the \( GTDOF \) control system can also be formulated by the \( SF \) decomposition introduced in section 2 and 3. Corresponding cost functions for the tracking and control properties can always be constructed by using the triangle inequality

\[
J_{\text{tracking}} \leq J_{\text{des}}^{\text{tr}} + J_{\text{real}}^{\text{tr}} + J_{\text{id}} = \|S_{\text{des}}^T\|_2 + \|S_{\text{real}}^T\|_2 + \|S_{\text{id}}^T\|_2
\]

\[
J_{\text{control}} \leq J_{\text{des}}^{\text{ct}} + J_{\text{real}}^{\text{ct}} + J_{\text{id}} = \|S_{\text{des}}^T\|_2 + \|S_{\text{real}}^T\|_2 + \|S_{\text{id}}^T\|_2
\]

The possible minimization of the second term will be discussed in the sequel.

The goal of this optimization step is to minimize the realizability loss \( J_x^{\text{real}} \) using optimal embedded filters \( G_x = G_x^{\text{opt}} \) attaining the influence of the invariant model factor \( \hat{P} \).

\[
G_x^{\text{opt}} = \arg \left\{ \min_{G_x} \|y_x R_x \left\{ 1 - G_x \hat{P} \right\} \| \right\}
\]

This task corresponds to the model matching approach (see Fig. 3) of control system design and different from the one given by (36). The optimal realizability degradation is considerably different for IS and IU processes. For the IS case \( \hat{P} = 1 \), so there is no optimization problem to be solved and the trivial selection \( G_x^{\text{opt}} = G_x = 1 \) can be used. The realizability degradation is zero now.

![Figure 3. Reduced form of \( H_2 \) optimality for \( S_x^{\text{real}} \)](image)

Assume the same reference model as in (37) and formulate the \( H_2 \) optimality for the cost function

\[
J_x^{\text{real}} = \|R_x \left\{ 1 - G_x B_\infty \right\} \|_2 = \left\| B^- \frac{B_x}{B_. A_s s^k} - B_\infty G_x B^- \right\|_2
\]

where \( B_- \) has the same meaning as previously. Thus \( B_- \) is stable and \( \left\| B_. B^- \right\|_2 \) is a stable norm-preserving inner function. Taking the usual decomposition

\[
\frac{B_x}{B_. A_s s^k} = \frac{R}{B_.} + \frac{K}{A_s s^k} = \frac{R A_s s^k + K B_\infty}{B_. A_s s^k}
\]

where in the right hand side \( R/B_. \) is the non-causal and \( K/A_s s^k \) is the causal part, consequently (54) becomes

\[
J_x = \left\| \frac{B_. s^k R}{A_s s^k} + \frac{K}{A_s s^k} B_\infty \right\|_2
\]

Following the orthogonality principle the \( H_2 \) optimal \( G_x \) can be obtained by making the causal part zero, i.e.

\[
\frac{K}{A_s s^k} = 0
\]

which results in

\[
G_x = \frac{K}{B_.}
\]

\( R \) and \( K \) can be obtained from the special Diophantine equation
The minimum of the cost function formally is given by (45), where $\mathcal{R}$ is obtained from (59) now. The $\mathcal{H}_2$ optimal regulator using (29) and (42) is

$$C_\ast = \frac{\mathcal{K}A}{\mathcal{B}_s (\mathcal{B}_s' - \mathcal{K}P)}$$

(60)

Let us investigate the product

$$G_x \mathcal{B}_s = \frac{\mathcal{K} \mathcal{B}_s}{\mathcal{B}_s' - \mathcal{K}P} = 1 - \frac{\mathcal{A}_\ast \mathcal{K}}{\mathcal{B}_s' B'_s} s^k$$

(61)

where (59) was used, it is easy to see that $G_x \mathcal{B}_s \mid_{s=0} = 1$, providing integrating regulator, can be obtained if, and only if $k \geq 1$. Again: the original formulation of the $\mathcal{H}_2$ optimality of $\|y_s\|_2$ using the operator norm of the SF can not provide an integrating regulator. This is why the $y_{sx}(s) = s^{-k}$ form excitation was assumed and the induced norm $\|y_s\|_2$ was used in the optimization.

Example 2.

Solve the previous example searching the $\mathcal{H}_2$ optimality of $\|y_s\|_2$. The solution of the Diophantine equation (59) for $k = 0$ gives

$$\mathcal{R} = \frac{2T}{T_w + T} = \frac{2}{1 + a} ; \mathcal{K} = \frac{T_w + T}{T_w - T} = \frac{a + 1}{a - 1} ; a = T_w/T$$

(62)

The optimal filter is now

$$G_x = \frac{T_w + T}{T_w - T} \frac{1}{1 + sT} = \frac{a + 1 + 1}{a - 1 + sT}$$

(63)

It is easy to check that the gain of $G_x \mathcal{B}_s$ is not one now, so the corresponding regulator is not integrating.

Solving the problem for $k = 1$ gives

$$\mathcal{R} = \frac{2T^2}{T_w + T} = \frac{2T}{1 + a} ; b = \frac{2T}{T_w + T} = \frac{2}{1 + a}$$

(64)

$$\mathcal{K} = 1 + T_x s = 1 + 2bT_{w} s$$

and the optimal filter is

$$G_x = \frac{1 + T_x s}{1 + sT}$$

(65)

It is easy to check that the gain of $G_x \mathcal{B}_s$ equals to one, so the corresponding regulator is an integrating one.

7. CONCLUSIONS

The authors believe that the relatively easy and reasonably optimal solution of a generally very sophisticated control problem strongly depends on the proper decomposition of the original paradigm. These decompositions correspond to a natural control engineering practice, too, where the best reachable design goal and the way how to obtain it appear in a generally iterative sequential procedure.

The paper investigated the $\mathcal{H}_2$ optimality of the SF in a $\text{GDTOF}$ control system and the realizability loss in the decomposed form of the SF. The previous, classical optimality does not depend on the design goal, therefore the second one fits better to the practical requirements. The optimization provides integrating regulator, iff an $y_{sx}(s) = s^{-k}$ form excitation is assumed with $k \geq 1$.

The results can be easily applied for discrete time systems where $\mathcal{B}_s$ contains the unstable zeros of $\mathcal{B}_s$ mirrored on the unit circle and $y_{s}(z) = (z^{-1})^{-k}$.

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