Nonlinear System Identification and Fault Detection using Hierarchical Clustering Analysis and Local Linear Models

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Abstract—This paper discusses the use of unsupervised learning and localized modeling to identify nonlinear dynamical systems from empirical data. A finite-order nonlinear autoregressive (AR) model is constructed to capture the system dynamics. The embedded input space for the nonlinear AR model is partitioned into overlapped regions that are fine enough so that localized modeling techniques, such as local linear modeling, can approximate system dynamics well in each region. Subsequently, unsupervised learning, such as hierarchical clustering analysis, is used for partitioning the embedded input space to achieve the tradeoff between the model complexity and the approximation error. The performance of the proposed nonlinear system identification is evaluated on two numerical examples: (i) time series prediction; (ii) identification of SISO system. Intelligent fault detection scheme is designed based on the identified linear models. Simulation results demonstrate that the proposed approach can capture the nonlinear system dynamics well and correctly detect the faults.

I. INTRODUCTION

In many cases involving complex nonlinear systems, it is very difficult or impossible to derive dynamic models based on all the physical processes involved. Dynamical system identification provides an alternative way to building mathematical models of nonlinear systems to approximate system dynamics [1]. Typically, a certain parameterized linear or nonlinear model is constructed to resemble the original physical system. The parameters should be learned to minimize errors between estimated (or predicted) and actual system dynamics. The resulting model can be used as a tool for system analysis, simulation, prediction, monitoring, diagnosis, and controller design.

The dynamical neural network paradigm is a promising tool for the identification and control of a variety of nonlinear dynamical systems [2, 3, 4, 5]. For example, feedforward neural networks have been successfully applied to the prediction and modeling of chaotic time series, nonlinear filtering, and input-output modeling of nonlinear processes. Due to the universal mapping characteristics, enhanced supervised feedforward architectures with short-term memory mechanisms, can approximate arbitrarily well any continuous input-output mapping [6, 7].

The concept of multiple models and switching between models has been an area of interest in nonlinear system identification and control. It has been widely believed that most of dynamical systems in practice exhibit non-linear behaviors. The traditional approach – statistical and based on the linear model, would be no longer able to capture dynamics of the nonlinear system. This gave rise to the notion of using multiple models to represent nonlinear system dynamics over the entire operating regime. In this approach, a set of models is designed to represent possible system behaviors. According to strategies to derive multiple models, the multiple modeling can be categorized as: (1) physical model-based; (2) learning theory-based. Multiple Kalman filter-based modeling has been applied to improve the accuracy in state estimation and control problems [8], and each possible system dynamics is represented by a Kalman filter. Neural network-based multiple modeling has been applied for the aircraft failure detection and identification [9]. Self-organizing map (SOM) based multiple models have been been proposed for modeling and controlling non-linear plants [10], and the global dynamics is approximated by a preset number of local linear models. However, it is difficult to select the appropriate number of Kalman filters or the size of SOM map in advance without any a prior information available, and inappropriate selection of model complexity may cause over-approximation or under-approximation of the original system dynamics.

Inspired by the idea of multiple models, we propose a method using hierarchical clustering analysis to identify the model complexity (or number of models), and local modeling to approximate dynamics of the nonlinear system. First, the unknown system dynamics is represented by a finite-order nonlinear AR model. The input to the nonlinear AR is embedded using the delayed input and output sequences. The system dynamics is divided into a set of regions according to distinctive behaviors, where a linear system is used to model the dynamics of each region locally. The global dynamics is approximated by the set of local linear models. Localized modeling has the advantage of representing local dynamics over traditional global modeling methods [11]. The overall fitting to the state space trajectory using local modeling may be more accurate when compared to a single global nonlinear model. In this paper, the hierarchical clustering analysis [12] is utilized to cluster the dynamic state space into different regions. The overall state space is represented by several clusters. The local linear model is derived from the local cluster using the least square fitting method. At any time instant, the similarity measure between the current state and clusters is calculated, and the model from the winning cluster is chosen to represent system dynamics. The architecture of the proposed nonlinear system identification is illustrated in...
The rest of paper is organized as follows. Section II introduces global and local modeling of nonlinear system dynamics. Section III briefly describes the hierarchical clustering analysis, and the segmentation of system dynamics into different regimes. The local linear modeling is used to approximate the local dynamics. The MMSE-based fault detection scheme is presented in Section IV. Section V presents simulation results from the application of the proposed approach for problems of time series prediction, system identification and fault detection for nonlinear SISO. Section VI summarizes and gives directions for future research.

II. GLOBAL AND LOCAL DYNAMIC MODELING

Given experimental time series data, measurements or observations, identification of nonlinear dynamical system is to quantify the system that created the time series. Generally, the nonlinear dynamical system can be represented by a set of $K^{th}$ ordinary nonlinear differential equations as

$$\frac{d}{dt} \mathbf{s}(t) = \Phi(\mathbf{s}(t)), \quad (1)$$

where $K$ is the unknown dimension of the nonlinear dynamical system, $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]$ is a vector of system states and $\Phi(\cdot)$ is called the vector field.

When working with experimental data produced by the nonlinear dynamical system, knowledge of the nonlinear differential equations (1) is unavailable [13]. The observation of system outputs and inputs is the only information from the unknown dynamical system. The key to the nonlinear system identification is what we can infer about the system dynamics from the observation of an output and/or input time series. It is shown experimentally [14] and proven [15] that an embedded sequence of sampled observations

$$\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-N+1)],$$

can be used to create a trajectory of the state space. The trajectory preserves the dynamical invariants (correlation dimension and Lyapunov exponents) of the original dynamical system. According to Taken’s embedding theorem, a mapping function $F(\cdot) : R^N \to R^N$ exists that transforms the current reconstructed state $\mathbf{x}(n)$ to the next state $\mathbf{x}(n+1)$, i.e.

$$\mathbf{x}(n+1) = F(\mathbf{x}(n)) \quad (2)$$

or

$$\begin{bmatrix} x(n+1) \\ \vdots \\ x(n-N) \end{bmatrix} = F \left( \begin{bmatrix} x(n) \\ \vdots \\ x(n-N+1) \end{bmatrix} \right),$$

where the value of $N$ is chosen to meet the conditions of Taken’s embedding theorem as

$$N > 2K.$$ 

As we can see, equation (2) contains several nonlinear filters and a nonlinear predictor. The predictive mapping, $R^N \to R$, is the basis of modeling. The predictor can be expressed as

$$x(n+1) = f(\mathbf{x}(n)), \quad (3)$$

which is actually a nonlinear autoregressive model of the input time series. The nonlinear filters can be derived based on the predictive mapping. Hence, the mapping function $F$ can be obtained from the predictor by simple matrix operations. If the conditions of Taken’s embedding theorem are satisfied, the mapping can capture the characteristics of the unknown dynamical system. Thus, the task of nonlinear dynamical system identification is equivalent to estimate the mapping function. However, unlike the linear case, there is no algorithmic way to determine the mapping function. The framework of function approximation is an alternative way to estimate the mapping function.

A function space is defined based on the set of basis functions, $\Psi = \{\varphi_1(x), \varphi_2(x), \ldots, \varphi_N(x)\}$. The mapping function $F(\mathbf{x}(n))$ can be approximated by

$$\hat{F}(\mathbf{x}(n), \mathbf{w}) = \sum_{i=1}^{N} w_i \varphi_i(\mathbf{x}(n)), \quad (4)$$

where $\mathbf{w}$ is the weighting parameter vector. The approximation error is defined as

$$J(\mathbf{w}) = \sum_{\mathbf{x}} \text{dis}(\mathbf{x}(n+1) - \hat{F}(\mathbf{x}(n), \mathbf{w})), \quad (5)$$

where $\text{dis}(\cdot)$ is a metric to measure the fitness of the approximated function, usually taken as the Euclidean distance. The optimal weighting vector, $\mathbf{w}^*$ is determined to minimize the error function as

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} J(\mathbf{w}).$$
As mentioned early, the nonlinear system dynamics normally consists of different regimes with distinct behaviors. Thus, instead of using a single mapping \( F(\cdot) \) to approximate the reconstructed state space (global modeling), the mapping is decomposed into a family of maps, \( \{ F_i(\cdot), i = 1, \ldots, c \} \), where \( c \) denotes the number of different dynamic regimes. Each individual map captures the corresponding regime dynamics, and is estimated using the reconstructed state in that regime. This mapping estimation is called as local modeling. The overall mapping is then a concentration of local mappings

\[
\hat{F}(x(n)) = \bigcup_{i=1, \ldots, c} \tilde{F}_i(x(n)),
\]

Similarly, the overall predictive mapping can be described by local predictive mappings

\[
\tilde{f}(x(n)) = \bigcup_{i=1, \ldots, c} \tilde{f}_i(x(n)),
\]

where \( \tilde{f}(\cdot) \) and \( \tilde{f}_i(\cdot) \) are the global and local predictors respectively.

Assuming that individual dynamic regime is fine enough and the underlying state evolution is sufficiently smooth, the local predictive mapping function \( \tilde{f}_i(x) \) in the vicinity of \( x(n) \) can be approximated by the first few terms of its multidimensional Taylor series expansion

\[
\tilde{f}_i(x) = \tilde{f}_i(x(n)) + \nabla \tilde{f}_i(x(n))(x - x(n)) + \cdots \approx a_i^T x + b_i,
\]

where the linear coefficient vector \( a \) and scalar \( b \) are estimated from reconstructed states in the \( i \)th dynamic regime. Thus, the very complicated estimation of the mapping function \( F(\cdot) \) is simplified to search for the local dynamic regimes and subsequently use linear modeling to fit the local state space.

Before we proceed to design an unsupervised learning for searching of local dynamic regimes, nonlinear AR models for time series prediction and SISO system identification are briefly described. Consider the scalar time series data is produced by

\[
\frac{d}{dt} x(t) = G(x(t)),
\]

where \( G(\cdot) \) is a nonlinear function. The data sampling period is \( \tau \). The nonlinear AR model of order \( n_x \), which captures the nonlinear system dynamics, is expressed as

\[
x(n + 1) = f(x(n), x(n - 1), \ldots, x(n - n_x + 1)),
\]

where \( f(\cdot) \) is the nonlinear mapping function, \( R^{n_x} \rightarrow R \). For the purpose of time series prediction, the input to the unsupervised learning is defined as

\[
x_{in}(n) = [x(n), x(n - 1), \ldots, x(n - n_x + 1)],
\]

\[
x_{out}(n) = x(n + 1).
\]

For the purpose of SISO system identification, two time series are involved, one is the system input variable, \( u(t) \) and the other is the output observation, \( z(t) \). The goal is to estimate the next output based on current and previous values of the input and output. Mathematically, the nonlinear AR model for SISO system can be stated as

\[
x(n + 1) = f(x(n), u(n)),
\]

where \( x(n) \) and \( u(n) \) are embedded output and input sequences as

\[
x(n) = [x(n), x(n - 1), \ldots, x(n - n_x + 1)],
\]

\[
u(n) = [u(n), u(n - 1), \ldots, u(n - n_u + 1)],
\]

where \( n_x \) and \( n_u \) are the orders of the nonlinear AR model. The augmented input to learn the local dynamic regimes is defined as

\[
x_{in}(n) = [x(n), u(n)],
\]

\[
x_{out}(n) = x(n + 1).
\]

The prediction or estimation residual is defined as

\[
e(n + 1) = x(n + 1) - \hat{x}(n + 1).
\]

In order that the nonlinear AR model can capture the nonlinear system dynamics, the order of the nonlinear AR model should meet the conditions of Taken’s embedding theorem. However, it is quite difficult to select the appropriate model order in advance without any a priori knowledge of the dimension of the nonlinear dynamical system. Assume that several sets of fully tuned local predictive mappings are obtained from the underlying nonlinear AR models with different orders, the appropriate nonlinear AR model to represent the original system is the one with minimal estimation or prediction error that can be evaluated using the mean squared error metric. The mean squared estimation error is defined as the function of orders of the nonlinear AR model,

\[
MSE(n_x, n_u) = \frac{1}{N} \sum_n \| x(n) - \hat{x}(n) \|^2.
\]

The selection of nonlinear AR model with optimal orders is the one with minimum mean squared estimation error as shown in Figure 2.
III. CLUSTERING ANALYSIS OF SYSTEM DYNAMICS

The system dynamics can be partitioned into different regimes according to the underlying distinct behaviors. It is natural to organize different regimes following the rule that reconstructed states with large similarity shall be grouped into the same regime. It is also desirable that the number of regimes can be determined during the partitioning process. The hierarchical clustering analysis proposed in [12] is quite suitable for the partitioning of the system dynamics without any a priori system information available. Before the clustering process, the input state is constructed using current system output and past information as \( \{x(n+1), [x(n), u(n)]\} \). This augmented pair contains the information to help us observe local instances of the state. If enough of these pairs are available to cover all of the state space, the nonlinear system dynamics can be identified.

The hierarchical clustering creates a tree-structured clustering, where similar states are grouped together to form a cluster. The clustering result very much depends on the similarity measure used. In this paper, proximity matrix is constructed using the Euclidean distance metric.

Given a set of \( N \) states, \( \{x_i \in R^d\} \), the proximity matrix \( D \in R^{N \times N} \), has nonnegative entries. The diagonal elements are set to be equal to one. The off-diagonal elements are computed as

\[
D_{ij} = \Phi(\|x_i - x_j\|) \quad \forall i \neq j, \tag{12}
\]

where \( \Phi \) is monotonic nonlinear transfer function from \( R^d \) to \( R \), and is chosen as the Gaussian function,

\[
\Phi(\|x_i - x_j\|) = \exp(-\frac{\|x_i - x_j\|^2}{2\sigma^2}), \tag{13}
\]

where \( \sigma \) is chosen according to the span of the state space. The similarity measure between two states is constrained from 0 to 1; and the more similar two states are, the larger similarity they have. As we can see, the proximity matrix is symmetric. The detailed clustering process is referred to the hierarchical clustering algorithm in [12].

After the state space being clustered, a sparse matrix, \( C_L \in R^{N \times N} \) is created, in which the nonzero elements in each row represents the states in the same sub-cluster. The matrix \( C_L \) has the form as

\[
C_L = \begin{pmatrix}
1 & i & j & \cdots & k & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
l & m & h & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\end{pmatrix}.
\]

The value of each nonzero element in matrix \( C_L \) represents states in the series \( \{x_i\}_{i=1}^{N_c} \). The total number of cluster is denoted as \( c \), which is equal to the number of nonzero elements in the first column of the matrix \( C_L \). By examining the matrix \( C_L \), we can obtain the number of clusters (system dynamic regimes), states within each cluster. Generally we will have \( c \ll N \), which means that the overall system dynamics can be represented by several local dynamic regimes.

The hierarchical clustering analysis preserves topological relationships in the input state space by grouping the similar states together to form a cluster. The linear model to approximate the local dynamics can be learned in a supervised way. The least square fit is applied for the estimation of the local linear model coefficients, \( a_i \) and \( b_i \). In order to avoid the singularity problem, only kept for the system dynamic approximation are those clusters with number of states larger than or at least equal to the orders of nonlinear AR model. The training process comprises: (i) determination of the nonlinear AR model orders; (ii) formation of the augmented input state space from the observations; (iii) searching of the local dynamic regimes using hierarchical clustering analysis; (iv) fitting the local state space using linear modeling. During the testing process, the similarity between the instant input and clusters are computed, and the instant system dynamics is described by the local linear model of the winning cluster.

IV. MMSE-BASED FAULT DETECTION SCHEME

Often the nonlinear system operates under different environmental conditions or different operating modes. It is of great importance to detect and identify these various operating modes in order to guarantee the mission success. In this paper, a simple and reliable fault scheme is designed based on the minimum mean squared error estimation. The dynamics of nonlinear system is compared with a bank of known or identified models. These models represent the possible system operating conditions. By comparing residuals from each model, the one with the minimum estimation error is selected to represent the nonlinear system. A “moving window” of size \( M \) is applied for the calculation of estimation error, which the \( M \) most recent residuals are examined to determine whether the dynamics from the models differs significantly from the dynamics of the nonlinear system. The number \( M \) is a design parameter. The windowed mean squared estimation error for the \( j^{th} \) model at time \( i \) is defined as

\[
\text{MSE}_j(i) = \frac{1}{M} \sum_{l=i-M+1}^{i} \| x(l) - \hat{x}(l) \|^2.
\]

Figure 3 presents the system operating mode detection and identification scheme. A bank of known or identified models
runs in parallel, with each of them represents a possible system operating mode. The current system operating mode is determined to be the one with minimum mean squared estimation error.

\[ k = \arg \min_j \text{MSE}_j \quad j = 1, \cdots, K \quad (15) \]

V. PERFORMANCE EVALUATION

To assess the effectiveness of the proposed approach we conduct experiments on the problems of time series prediction and SISO system mode detection and identification.

A. MODE DETECTION AND IDENTIFICATION OF SISO NONLINEAR SYSTEM

The SISO nonlinear dynamic system to be identified is described by the first order difference equation,

\[ x(n+1) = \frac{29}{40} \sin \left( \frac{16u(n) + 8x(n)}{3 + 4u^2(n) + 4x^2(n)} \right) + \frac{2}{10} u(n) + \frac{2}{10} x(n). \quad (16) \]

where \( x(0) = 0 \) and \( n \) is the time index. The input signal is \( u(n) = 0.5 \sin(0.25n) \).

5000 data samples are collected, the first 1000 data samples are used for the training purpose to identify the nonlinear system, and the rest are for the testing. The orders of the nonlinear AR model is determined as \( n_x = 2 \) and \( n_u = 1 \). The output signal from the identified model and real model is shown in Figure 4. As we can see, outputs of the identified model follow the trace of the real model outputs. The estimation error is quite small, which means the nonlinear dynamics of SISO system can be captured using the proposed nonlinear system identification scheme.

It is assumed that the system operating mode of nonlinear system (16) changed after \( t = 400s \), where the constant \( \frac{29}{40} \) (mode 1) changed to \( \frac{16}{20} \) (mode 2). This simulates the effect of damage or failure in the nonlinear system. Each operating mode is identified using the proposed nonlinear system identification scheme. The residual from each corresponding model is calculated. The window size, \( M \) is chosen as 100.

The windowed mean squared estimation error for each mode is shown in Figure 6. The system mode decision signal is shown in Figure 5. It tells in which mode of operation is the actual plant. One can see that it actually discovers that system mode is switched at \( t = 400s \).

B. TIME SERIES PREDICTION

The proposed approach for nonlinear system identification is tested on the problem of time series prediction in Lorenz system, which exhibits chaotic dynamics with large Lyapunov exponent for \( \alpha = 10, \beta = \frac{8}{3} \) and \( \gamma = 28 \). The Lorenz equations are

\[ \dot{x} = \alpha(y - x), \]
\[ \dot{y} = x(\gamma - z) - y, \]
\[ \dot{z} = xy - \beta z. \]

The time series data is sampled at 10Hz. The embedding dimension is chosen as 5, and so the dimension of the state input during the learning process is 18 so that all three states can be predicted concurrently. 5000 data samples are generated. The system dynamics is identified with the first 1000 data samples. Figure 7 shows the state prediction and the prediction error using approximated prediction model. As we can see, the approximated model can track the chaotic dynamics of the Lorenz system and provides quite accurate state prediction.
VI. CONCLUSIONS AND FUTURE WORKS

We proposed the approach to identifying the nonlinear system dynamics based on unsupervised learning and local linear modeling. Given the experimental time series data, the proposed approach applies the nonlinear AR model to approximate the system dynamics. The reconstructed state space is augmented using current system output and observation history. The hierarchical clustering analysis is used to partition the state space into several clusters, where each cluster contains the information of the local dynamics. The linear modeling technique is used to fit states locally. The overall system dynamics are approximated using a set of local linear models instead of a single complicated global model. The identified models can be used to design the system operating mode detection and identification scheme. The windowed MMSE estimator is used to compare the system operating mode can be identified. Experimental results demonstrate the effectiveness of the proposed approach for problems of time series prediction and SISO nonlinear system identification. However, the local linear modeling may break the smoothness of the original model. The spikes in the state prediction error reveal the model discontinuity caused by the local linear modeling. In the future work, smoothed local modeling algorithms will be investigated and applied for the local dynamic approximation, and the proposed approach for nonlinear dynamical system identification will be applied for the nonlinear controller design and machinery diagnostics.

REFERENCES


