Adaptive Neural Network Tracking Control for
A Class of MIMO Nonlinear Systems with
Measurement Error

Artemis K. Kostarigka, George A. Rovithakis
Dept. of Electrical & Computer Engineering
Aristotle University of Thessaloniki
54124, Thessaloniki, Greece

Abstract—An adaptive neural network tracking controller is designed, capable of stabilizing multi-input multi-output nonlinear, affine in the control dynamical systems with measurement error. Uniform ultimate boundedness of the tracking error of the actual system state is guaranteed, as well as boundedness of all other signals in the closed loop. Possible division by zero is avoided with the use of a novel resetting procedure, capable of guaranteeing the boundedness away from zero of certain signals.

I. INTRODUCTION

Neural network theory has emerged rapidly during the last decade, challenging many scientists in the area of control, since it provided the means to manipulate highly nonlinear systems, functioning in volatile environments.

After the first insight of Narendra and Parthasarathy [1], many scientific works have focused on the use of neural networks for controlling highly uncertain and possibly unknown nonlinear dynamical systems. The key idea was to exploit their universal approximation capabilities, substituting every unknown nonlinearity by a neural network model of known structure but containing a number of unknown parameters (synaptic weights) plus a modeling error term. The unknown synaptic weights may appear both linearly or non-linearly, thus popularizing the neural network framework as one of the most powerful control design tool. Representative works include [2]–[17].

In the case of full state measurement the problem of controlling highly uncertain systems has found satisfactory solution. Unfortunately, this is not the case when we have erroneous state measurement. The majority of existing techniques are based on the complete knowledge of system nonlinearities and the absence of any modeling error terms. Only recently the problem of adaptively stabilizing unknown nonlinear systems with measurement error has been considered. Existing results are constrained to certain system classes such as linear [18], partially linear [19], feedback linearizable [20] or nonlinear systems represented by piecewise interpolation of several linear models [21]. Furthermore, in [13], the neural adaptive regulation under measurement noise problem for affine in the control nonlinear dynamical systems, has been treated under a restrictive regressor decomposition assumption.

In this paper we generalize [13] by considering the tracking problem and by relaxing the regressor decomposition assumption. In this respect, we design an adaptive neural network controller, capable of stabilizing multi-input multi-output nonlinear affine in the control dynamical systems with measurement error. More specifically, by making use of Lyapunov stability theory, uniform ultimate boundedness of the tracking error of the actual system state is guaranteed, as well as boundedness of all other signals in the closed loop. An important aspect of the proposed control scheme is that knowledge of an upper bound on the measurement error is not required. Thus, the proposed controller can be applied to a sufficiently general class of nonlinear systems (those that are affine in the control) admitting a wide class of output error signals.

To avoid possible division by zero, the developed controller is equipped with a novel resetting procedure, capable of guaranteeing the boundedness away from zero of certain signals.

Finally, algebraic relations including design constants values and measurement error bounds are provided, to guarantee the validity of the neural network approximation capabilities at all times.

The paper is organized as follows: In Section II we review basic definitions and preliminary results. Section III is focused on designing an adaptive neural network tracking controller for nonlinear control systems with measurement error. Simulation results are provided in Section IV. Finally, we conclude in Section V.

II. DEFINITIONS AND PROBLEM STATEMENT

Consider the system

\[ \begin{align*}
\dot{x} &= f(x, u), \quad x \in \mathbb{R}^n, u \in \mathbb{R}^m \\
y &= h(x, u), \quad y \in \mathbb{R}^l
\end{align*} \]  

(1)

**Definition 1:** [22] The solutions of (1) are **uniformly ultimately bounded (u.u.b.)** (with bound B) if there exists a $B > 0$ and if corresponding to any $a > 0$ and $t_0 \in \mathbb{R}^+$, there exists a $T = T(a) > 0$ (independent of $t_0$) such that $|x(t; t_0, x_0)| < B$ for all $t \geq t_0 + T$.

A. Linear in the weights neural networks

In the present work we shall be using linear in the weights neural networks of the form

\[ z^T = w^T S(v) \]  

(2)

where $v \in \mathbb{R}^{n_2}$ and $z \in \mathbb{R}^{n_1}$ are the neural net input and output respectively, $w$ is an $L-$dimensional vector of...
synaptic weights and $S(e)$ is an $L \times n_1$ matrix of regressor terms. The regressor terms may contain high order connections of sigmoidal functions [14], [15], radial basis functions (RBFs) with fixed centers and widths [23–25], shifted sigmoids [26], [27], thus forming high-order neural networks (HONNs), RBFs and Shifted Sigmoidal Neural Networks, respectively. Another class of linear in the weights neural nets is the CMAC network which mainly uses B-splines in $S(e)$ [28].

An important and well known property shared among the aforementioned neural approximating structures is the following (see also the references above):

**Density Property:** For each continuous function $f : \mathbb{R}^m \rightarrow \mathbb{R}$, and for every $\epsilon_0 > 0$, there exist an integer $L$ and an optimal synaptic weight vector $w^* \in \mathbb{R}^L$ such that $sup_{e \in \mathbb{R}^m} \left| f^T(e) - w^T S(e) \right| \leq \epsilon_0$, where $\Omega \subset \mathbb{R}^n$ is a given compact set.

This property implies that if the number of the regressor terms $L$ is sufficiently large, then there exist weight values $w^*$ such that $w^T S(e)$ can approximate $f^T(e)$ to any degree of accuracy, in a compact set.

**B. Problem Statement**

Let us consider the system:

$$\dot{x}(t) = f(x) + g(x) u(t)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input while $f(x), g_i(x), i = 1, 2, \ldots, m$, with $g(x) = [g_1(x) \ g_2(x) \ \ldots \ g_m(x)]$ are continuous, locally Lipschitz and unknown vector fields. The state measurement is corrupted by an unknown, bounded, time varying and continuous measurement error signal $\eta(t) \in L_\infty$, thus making $x(t)$ not available for control.

Define the tracking error as $e = x - x_r$, where $x_r(t)$ is a bounded reference trajectory with $\dot{x}_r(t)$ also bounded. Differentiating $e$ (3) also yields:

$$\dot{e} = f(x) + g(x) u(t) - \dot{x}_r(t)$$

**Assumption 1:** For the error system (5), the solution can be forced to be uniformly ultimately bounded with respect to an arbitrarily small neighborhood of $e = 0$.

Owing to Assumption 1, there exists a control Lyapunov function $V(e) : \mathbb{R}^n \rightarrow \mathbb{R}_+$ such that

$$\alpha_1 ||e|| \leq V(e) \leq \alpha_2 ||e||$$

with $\alpha_1, \alpha_2 \in K_\infty$, and a control input $u \in \mathbb{R}^m$ such that

$$\dot{V} = \frac{\partial V}{\partial e} \left[ f(x) + g(x) u - \dot{x}_r(t) \right] \leq 0, \text{ } \forall e \in \mathcal{E}$$

where $\mathcal{E} = \{ e \in \mathbb{R}^n : ||e|| > \varepsilon \}$ with $\varepsilon > 0$. From (3), (4) we obtain:

$$\dot{V} = \frac{\partial V}{\partial e} \left[ f(x) + g(x) u - \dot{x}_r(t) \right]$$

$$= \frac{\partial V}{\partial e} \left[ f(u - \eta) + g(u - \eta) u - \dot{x}_r(t) \right]$$

$$= \frac{\partial V}{\partial e} \left[ f(u - \eta) - \dot{x}_r(t) \right] + \frac{\partial V}{\partial e} g(u - \eta) u$$

Functions $F(y, \eta, e, \dot{x}_r)$ and $G(y, \eta, e)$ can be trivially decomposed as follows:

$$F(y, \eta, e, \dot{x}_r) = F_1(y, \dot{x}_r) + F_2(y, \eta, e, \dot{x}_r)$$

$$G(y, \eta, e) = G_1(y) + G_2(y, \eta, e)$$

where $F_1(y, \dot{x}_r)$ and $G_1(y)$ contain only measurable quantities. Employing the inequality:

$$\gamma_3(\eta||) = \gamma_3(||u - x_r - \eta||) \leq \gamma_3(\eta||) + \gamma_32(||y - x_r||)$$

where $\gamma_31, \gamma_32 \in K_\infty$ functions, as well as the assumption (which will be proved in the sequel), that $u(t) \in L_\infty$, the components also containing the unmeasured quantities $\eta, e$ can be manipulated as follows:

$$F_2(y, \eta, e, \dot{x}_r) + G_2(y, \eta, e)|u| \leq [F_2(y, \eta, e, \dot{x}_r)] + [G_2(y, \eta, e)|u|]$$

where $\bar{u}$ denotes the constant but unknown upper bound on $|u(t)|$ and $\gamma_1(\cdot)$, $i = 1, \ldots, 4$ are $K_\infty$ functions. Using (9), (10) and (12), equation (8) becomes

$$\dot{V} \leq F_1(y, \dot{x}_r) + \gamma_1(y) + \gamma_4(\dot{x}_r)$$

$$+ \gamma_7(e) + \gamma_8(y - \dot{x}_r)$$

Since $F_1(y, \dot{x}_r)$, $G_1(y)$, $\gamma_1(y)$, $\gamma_4(\dot{x}_r)$ and $\gamma_8(y - \dot{x}_r)$ are considered unknown, the idea is to approximate them by suitable neural approximators. More specifically, it turns out that there exist continuous functions $\omega_i(\cdot)$, $i = 1, \ldots, 5$ (denoting the approximation errors) and constant but unknown weight vectors $w_i^*$, $i = 1, \ldots, 5$ such that:

$$F_1(y, \dot{x}_r) = w_1^* S_1(y, \dot{x}_r) + \omega_1(y, \dot{x}_r)$$

$$G_1(y) = w_2^* S_2(y) + \omega_2(y)$$

$$\gamma_1(\dot{x}_r) = w_3^* S_3(\dot{x}_r) + \omega_3(\dot{x}_r)$$

$$\gamma_4(\dot{x}_r) = w_4^* S_4(\dot{x}_r) + \omega_4(\dot{x}_r)$$

$$\gamma_8(y - \dot{x}_r) = w_5^* S_5(y - \dot{x}_r) + \omega_5(y - \dot{x}_r)$$

From the Density Property it also follows that on a generic compact set $\Omega \subset \mathbb{R}^n$, the approximation errors can be suitably bounded as $||\omega_i(\cdot)|| \leq \varepsilon_i$, $i = 1, \ldots, 5$ where $\varepsilon_i$, $i = 1, \ldots, 5$ are unknown but small bounds.

**III. MAIN RESULTS**

Let us consider the Lyapunov function candidate

$$L(e, \tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{w}_4, \tilde{w}_5) = k V(e) + \frac{1}{2} \sum_{i=1}^{5} ||\tilde{w}_i||^2$$

where $V(e)$ is the robust Lyapunov function defined in Section II, $k > 0$ is a design constant and $\tilde{w}_i$, $i = 1, \ldots, 5$ are parameter errors defined as $\tilde{w}_i = w_i - w_i^*$, $i = 1, \ldots, 5$.
where \( \hat{w}_i, i = 1, \ldots, 5 \) are estimates of the unknown parameters \( w_i^*, i = 1, \ldots, 5 \).

The following theorem summarizes the main results of this work.

**Theorem 1:** Consider the error system (5) and the state feedback control law:

\[
\dot{u}(t) = -h(y, w_2) + \frac{1}{k} \hat{w}_1 S_1(y, \dot{x}) + \frac{1}{k} \hat{w}_2 S_2(y) [\gamma(y) - \frac{1}{k} \hat{w}_3 S_3(y)] + \frac{1}{k} \hat{w}_4 S_4(\dot{x}_r) + \frac{1}{k} \hat{w}_5 S_5(y - \dot{x})
\]

with the design constants \( k > 0 \), \( i = 1, \ldots, 5 \), guarantee that:

1) \( u(t) \) is a priori bounded \( (u(t) \in \mathcal{L}_\infty) \), provided that

\[
h(y, w_2) \neq 0 \text{ } \forall \text{ } t \geq 0.
\]

2) the tracking error \( e(t) \) is u.u.b.

**Proof:**

1) From (20) we obtain

\[
\dot{u}(t) \leq \frac{1}{k} \hat{w}_1 S_1(y, \dot{x}) + \frac{1}{k} \hat{w}_2 S_2(y) + \frac{1}{k} \hat{w}_3 S_3(y - \dot{x}) + \frac{1}{k} \hat{w}_4 S_4(\dot{x}_r) + \frac{1}{k} \hat{w}_5 S_5(y - \dot{x})
\]

Notice that \( S_1(y, \dot{x}), S_2(y), S_3(y - \dot{x}), S_4(\dot{x}_r), S_5(y - \dot{x}) \) are bounded by construction. The estimates \( \hat{w}_i, i = 1, 3, 4, 5 \) are generated by (26),(28) and (30) respectively, which are in a bounded input bounded output form. Thus \( \hat{w}_i, i = 1, 3, 4, 5 \) are also bounded. Furthermore \( h(y, w_2) \) is claimed bounded away from zero. Hence \( u(t) \in \mathcal{L}_\infty \). In a later stage we shall present a resetting mechanism which is imposed on \( \hat{w}_2 \) to guarantee its uniform boundedness as well as \( h(y, w_2) \neq 0 \text{ } \forall \text{ } t \geq 0 \).

2) Differentiating (19) along the solutions of (3) using (13) we have

\[
\dot{\gamma}_i(y) = k \gamma_i(y), \quad i = 1, 4, 32, \eta.
\]

Employing the neural network approximators (14)-(18), \( \dot{\gamma}_i(y) \) becomes

\[
\dot{\gamma}_i(y) = k \gamma_i(y) \quad i = 1, 4, 32, \eta
\]

where \( \gamma_i(y) = \gamma_i(\cdot), \quad i = 1, 4, 32, \eta \).

Owing to the Density Property and the boundedness of \( u(t) \) we may straightforwardly conclude that \( \omega(y, x, \dot{x}, u) \leq \epsilon \) with \( \epsilon > 0 \) an unknown but small constant. After adding and subtracting the term

\[
\dot{\gamma}_i(y) = k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

we obtain:

\[
\dot{\gamma}_i(y) \leq k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

choosing the update laws

\[
\dot{\gamma}_i(y) = k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

where \( \gamma_i(y) = \gamma_i(\cdot), \quad i = 1, 4, 32, \eta \).

Employing the neural network approximators (14)-(18), \( \dot{\gamma}_i(y) \) becomes

\[
\dot{\gamma}_i(y) = k \gamma_i(y) \quad i = 1, 4, 32, \eta
\]

where \( \gamma_i(y) = \gamma_i(\cdot), \quad i = 1, 4, 32, \eta \).

Owing to the Density Property and the boundedness of \( u(t) \) we may straightforwardly conclude that \( \omega(y, x, \dot{x}, u) \leq \epsilon \) with \( \epsilon > 0 \) an unknown but small constant. After adding and subtracting the term

\[
\dot{\gamma}_i(y) = k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

we obtain:

\[
\dot{\gamma}_i(y) \leq k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

choosing the update laws

\[
\dot{\gamma}_i(y) = k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

where \( \gamma_i(y) = \gamma_i(\cdot), \quad i = 1, 4, 32, \eta \).

Employing the neural network approximators (14)-(18), \( \dot{\gamma}_i(y) \) becomes

\[
\dot{\gamma}_i(y) = k \gamma_i(y) \quad i = 1, 4, 32, \eta
\]

where \( \gamma_i(y) = \gamma_i(\cdot), \quad i = 1, 4, 32, \eta \).

Owing to the Density Property and the boundedness of \( u(t) \) we may straightforwardly conclude that \( \omega(y, x, \dot{x}, u) \leq \epsilon \) with \( \epsilon > 0 \) an unknown but small constant. After adding and subtracting the term

\[
\dot{\gamma}_i(y) = k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

we obtain:

\[
\dot{\gamma}_i(y) \leq k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

choosing the update laws

\[
\dot{\gamma}_i(y) = k \gamma_i(y) + \dot{\gamma}_i(y) + \epsilon
\]

where \( \gamma_i(y) = \gamma_i(\cdot), \quad i = 1, 4, 32, \eta \).

Employing the neural network approximators (14)-(18), \( \dot{\gamma}_i(y) \) becomes

\[
\dot{\gamma}_i(y) = k \gamma_i(y) \quad i = 1, 4, 32, \eta
\]
with $\mu = ke + \sum_{i=1}^{5} \frac{b_i}{\delta_i} |w_i|^2$. Defining $h(y, w_2) = w_2 S_2(y)$ and considering the control law

$$u(t) = -h(y, w_2) - \frac{w_2^T S_1(y, x_r) + \frac{1}{\delta_2} w_2^T S_2(y) |y|}{|h(y, w_2)|^2} - \frac{\gamma \rho(y - x_r) |y|}{|h(y, w_2)|^2} - \frac{\gamma \rho(y - x_r) |y|}{|h(y, w_2)|^2}$$

where $\gamma(\cdot), \rho(\cdot)$ are $K, K_{\infty}$-functions respectively, (32) yields:

$$\dot{L} \leq -\gamma(\rho(y(t) - x_r(t))) + \gamma_\eta(y) + \mu$$

Taking advantage of the weak triangular inequality [29] $\gamma(\alpha + \beta) \leq \gamma(\alpha) + \gamma(\rho(\alpha - \Id)^{-1}(\beta))$, which is valid for any functions $\gamma \in \mathcal{K}, \rho \in \mathcal{K}_{\infty}$ and any nonnegative real numbers $\alpha, \beta$, we get

$$\gamma(|\mathcal{L}|) \leq \gamma\rho(|y - x_r|) + \gamma(\rho(\rho - \Id)^{-1}(\eta))$$

Hence, $-\gamma(\rho(y - x_r)) \leq -\gamma(|\mathcal{L}|) + \gamma(\rho(\rho - \Id)^{-1}(\eta))$ and (34) becomes

$$\dot{L} \leq -\gamma(|\mathcal{L}|) + \gamma(\rho(\rho - \Id)^{-1}(\eta)) + \gamma_\eta(y) + \mu$$

If $|\mathcal{L}(t)| \geq \gamma(\rho(\rho - \Id)^{-1}(\eta)) + \gamma(\mathcal{L})$ then $\dot{L} \leq 0$. Thus, the tracking error $e$ is uniformly ultimately bounded with respect to the set

$$\mathcal{E} = \{e \in \mathbb{R}^n : |e| \leq \gamma(\rho(\rho - \Id)^{-1}(\eta)) + \gamma(\mathcal{L})\}$$

A. Resetting Procedure

In this subsection we modify the resetting procedure initially proposed in [16] to guarantee that $h(y, w_2) = w_2 S_2(y) \neq 0, \forall t \geq 0$. Initially, the value of $w_2$ is chosen so that $|h(y, w_2)| > \delta_2$, where $\delta_2$ is a user defined constant. Let us assume that at time $t = t_r, |h(y, w_2)| = \delta_2$. The following modification on the weight $w_2$ that correspond to a nonzero $S_2(y)$ coordinate is proposed to ensure that $|h(y, w_2)| > \delta_2, t = t_r^+$:

$$w_2(t_r^+) = w_2(t_r^-) + \sum_{i=1}^{5} \frac{k_i}{2} |w_i|^2$$

where $S_2(y)$ is the $S_2(y)$ coordinate that multiplies $w_2$, $h_j(t_r^-)$ is the $j$ element of $h$ vector which includes $w_2$, and $\gamma$ is the sign function. Furthermore $\phi(\cdot)$ is a positive, bounded function sharing the property $\lim_{|\nu| \to \infty} \phi(\nu) = 0$ with $\nu$ being a positive integer denoting the number of times the resetting procedure has been activated. To guarantee that $|h_j(t_r^+)| > |h_j(t_r^-)|$ it suffices to show that $|h_j(t_r^+)| > |h_j(t_r^-)|$. After performing the aforementioned resetting procedure at the $t = t_r^+$ we have:

$$|h_j(t_r^+)| > |h_j(t_r^-)| + |S_2(y) \phi(\nu) > |h_j(t_r^-)|$$

Clearly, (38) implies $|h_j(t_r^+)| > \delta_2$ thus guaranteeing $h(y, w_2) \neq 0, \forall t \geq 0$. Then the control law (33) and the update laws (26)-(30) apply and thus the stability properties stated in Theorem 1 are maintained $\forall t \in [t_1, t_2]$ where $[t_1, t_2]$ is any interval in $[0, \infty)$ for which $t_r \notin [t_1, t_2]$. To further guarantee that the resetting procedure may not drive $w_2$ to infinity owing to possibly infinite resettings, we argue that since $w_2(t_0) \in \mathbb{L}_{\infty}$ and $w_2(t) \in \mathbb{L}_{\infty}, \forall t \in [t_1, t_2] \subset [0, \infty]$, it suffices

$$\sum_{i=1}^{\infty} \sum_{j=1}^{5} |S_2(y)| h(y, w_2)| \phi(\nu) \in \mathbb{L}_{\infty}.$$ However, this is true owing to $\phi(\nu)$ being bounded $\forall \nu \geq 0$ and $\lim_{\nu \to \infty} \phi(\nu) = 0$.

Remark 1: The value of $\delta_2$ should be carefully selected since it is strongly connected to the control signal magnitude. A small $\delta_2$ reduces the minimum achievable value of $|h(y, w_2)|$, which in turn enlarges $u$.

B. Guaranteeing Approximation Capabilities

The results presented are valid as long as $e \in \Omega \subset \mathbb{R}^n$ where $\Omega$ is the compact set in which the approximation capabilities of the linear-in-the-weights neural networks hold. To verify that $e \in \Omega, \forall t \geq 0$ we observe that there exist functions $\gamma(\cdot), \rho(\cdot)$ as well as design constants $k, k_i, i = 1, \ldots, 5$ to guarantee that $\mathcal{E} \subset \Omega$, for a given $\Omega$. Hence we argue that if we start with an initial condition $e(0) \in \mathcal{E}$, then owing to the uniform ultimate boundedness of $e$, $e(t) \in \mathcal{E} \subset \Omega, \forall t \geq 0$. If however $e(0) \in \Omega \mathcal{E}$, the following analysis holds:

$$\mathcal{L} \leq -\gamma(|\mathcal{L}|) + \gamma(\rho(\rho - \Id)^{-1}(\eta)) + \gamma(\mathcal{L}) + \mu$$

If $|\mathcal{L}(t)| \geq \gamma(\rho(\rho - \Id)^{-1}(\eta)) + \gamma(\mathcal{L})$ then $\mathcal{L} \leq 0$. Thus, the tracking error $e$ is uniformly ultimately bounded with respect to the set

$$\mathcal{E} = \{e \in \mathbb{R}^n : |e| \leq \gamma(\rho(\rho - \Id)^{-1}(\eta)) + \gamma(\mathcal{L})\}.$$
where \( a_1^{-1}(\cdot) \) is a \( \mathcal{K}_\infty \) function since \( a_1(\cdot) \) is a \( \mathcal{K}_\infty \) function and \( \mathcal{L}_0 = kV(0) + \sum_{i=1}^{5} \bar{w}_i(0)^2 \).

Equation (43) clearly shows that there exists a \( k \) for which \( \mathcal{E}_1 = \{ e \in \mathbb{R}^n : |e(t)| \leq a_1^{-1} \left( \frac{\bar{w}_b}{\bar{M}} \right), \mathcal{L}_0 > \frac{\bar{M}}{k} \} \subset \Omega \). Moreover, for the set \( \mathcal{E}_2 = \{ e \in \mathbb{R}^n : |e(t)| \leq a_1^{-1} \left( \frac{\bar{w}_b}{\bar{M}} \right), \mathcal{L}_0 \leq \frac{\bar{M}}{k} \} \), considering that \( \bar{M} = \gamma(\rho \circ (\rho - 1\{ \bar{M} \}) + \gamma_0(\bar{M}) + \mu \), there exists a pair of functions (\( \rho, \gamma \)) as well as constants \( k, k_i, i = 1, \ldots, 5 \), guaranteeing \( \mathcal{E}_2 \subset \Omega \) for \( \Omega \) given and for every bounded measurement error signal \( |\eta(t)| \leq \bar{\eta}_b \). Thus in any case \( e(t) \in \Omega, \forall t \geq 0 \).

IV. SIMULATION RESULTS

Let's consider the scalar system \( \dot{x} = 2x - x^3 + x^2 u \) with the task to track the bounded reference trajectory \( x_r(t) = 1 + 0.5 \sin(4t) \) while subjected to a time varying, continuous and bounded measurement error \( \eta(t) = 0.01 \sin(100t) \).

To implement the control and update laws derived in Section III we have used the following matrices of regressor terms:

\[
S_1(y, \dot{x}_r) = \begin{bmatrix} s_a(y) & s_b(y) \dot{x}_r & s_a(y)^3 & s_b(y)^3 \dot{x}_r^3 \end{bmatrix}, \quad S_2(y) = \begin{bmatrix} s_a(y) & s_b(y) \end{bmatrix}, \quad S_3(\eta) = \begin{bmatrix} s_c(\eta) \end{bmatrix}, \\
S_4(\dot{x}_r) = \begin{bmatrix} s_c(\dot{x}_r) \end{bmatrix}, \quad S_5(\eta - \dot{x}_r) = \begin{bmatrix} s_c(\eta - \dot{x}_r) \end{bmatrix}
\]

The parameters \( k, k_i, i = 1, \ldots, 5 \) have been selected as \( k = 0.1, k_i = 0.001, i = 1, \ldots, 5 \), while \( \gamma(\cdot) = 10(1 - e^{-10(\cdot)}) \). The initial state condition was set to \( x_0 = 0.2 \), while the initial network weights where initialized as follows:

\[
w_{10} = [-1 \ -1 \ -1 \ -1 \ -1], \quad w_{20} = [-10 \ -10], \quad w_{30} = [-1], \quad w_{40} = [-1], \quad w_{50} = [-1].
\]

Finally, concerning the resetting procedure, the following constants have been used: \( \delta_b = 0.001, \varepsilon = 1 \).
with $\varphi(\nu) = \frac{1}{\nu}$. The aforementioned HONN structure and design parameters were selected according to a trial and error procedure. Theorem 1 provides guidelines for a qualitative selection.

The performance of the proposed control scheme is demonstrated in Figure 1, where the state trajectory is plotted (solid line) along with the reference signal $x_r(t)$ (dashed line). Figure 2 shows the corresponding tracking error that is clearly decaying to a neighborhood of zero, exponentially fast. In Figures 3, 4 and 5 the $L_2$ error norm is plotted clearly demonstrating that improved performance may be obtained by the appropriate choice of $\rho(\cdot)$ and $\gamma(\cdot)$, as well as by decreasing $k$.

V. CONCLUSIONS

We have presented an adaptive neural network controller for a sufficiently wide class of multi-input multioutput, nonlinear systems, which achieves tracking performance in the presence of continuous and bounded measurement errors. The proposed adaptive scheme is based on the estimation of certain signals that constitute the derivative of an unknown Lyapunov function, through the use of linear in the weights neural networks acting as universal approximators. Validity of the control input is provided by a resetting scheme, that prohibits possible division by zero. Uniform ultimate boundedness of the tracking error of the actual state is guaranteed, as well as the boundedness of all other signals in the closed loop.

REFERENCES


Fig. 5. $L_2$– Error norm $||e(t)||_2$ for $\gamma(\cdot) = 10(1-e^{-10(\cdot)}), \rho(\cdot) = \sin h(10(\cdot)))$ (solid line : $k = 0.1$, $- - - : k = 0.05$, $- - - - : k = 0.01, \cdots ; k = 0.001$)