A Robust Neuro-Adaptive Congestion Control Scheme with Respect to Exogenous Disturbances and Delay

C. N. Houmkozlis and G. A. Rovithakis

Abstract—A novel neuro-adaptive congestion controller is presented, capable of achieving certain time-quality characteristics, by regulating the per packet round trip. The controller is implemented at the source and is proven robust against modeling imperfections, exogenous disturbances (UDP traffic) and delays (propagation, buffering). The proposed source control theoretic framework is comprised of three dominant modules: i) an on-line desired round trip estimator, whose main objective is to output reliable piecewise constant predictions of feasible (i.e., achievable) round trip times; ii) an adaptive and saturated transmission rate controller, capable of precise tracking the desired round trip time commands that are downloaded from the first module and iii) a future path congestion level estimator, which provides critical information to the aforementioned two modules.

I. INTRODUCTION

The introduction of new types of services in the Internet has underlined the importance of Quality of Service (QoS) even in congested network conditions. This has further motivated researchers to develop new mechanisms capable of guaranteeing Quality criteria [10], [11], while preventing network from congestion collapse and throughput from starvation. Such applications typically demand higher sensitivity to time constraints (i.e., delay), so as to cause the least possible degradation of quality of service (QoS) to the underlying users, silently implying the existence of a control algorithm capable of regulating the round trip time.

Current control theoretic approaches are mainly focused on designing link algorithms, implemented as congestion controllers in the routers [2], [6]. An alternative, yet less explored approach, is to design congestion controllers to operate in the source. In this respect, source controls the packet transmission rate, with the plant now being the network as seen from the source. Such schemes though possess certain critical issues that should be carefully addressed. Specifically: 1) A model for the plant (network) has to be devised, otherwise stability analysis is impossible, 2) Determining a desired and feasible system performance is certainly a non-trivial task, owing to the highly uncertain and dynamically varying nature of the Internet, 3) The controlled system should not be only stable, but additionally robustly stable against modeling imperfections, exogenous disturbances (e.g., UDP traffic) and delay, and 4) The output of any proposed control scheme should be saturated.

Source control theoretic works started to appear very recently. In [1] linear recursive system identification is used to model the round trip dynamics and consequently an adaptive TCP congestion control strategy is proposed.

A different scheme was proposed in [3], [4] involving a nonlinear neural network model to capture the round trip dynamics. A nonlinear adaptive controller to determine the per packet sending rate was developed, to track an on-line estimated desired per packet round trip. The congestion control scheme was proven robust against modeling imperfections. However, the no delay, no disturbance (uncontrollable traffic) assumptions, restrict severely the applicability. In this paper we extend our previous works in the direction of relaxing the no delay, no uncontrollable traffic assumptions, while addressing the aforementioned critical issues (1)-(4).

The proposed congestion control scheme guarantees the uniform ultimate boundedness of the tracking error with respect to an arbitrarily small neighborhood of zero, plus the uniform boundedness of all other signal in the closed loop. Moreover, our controller is proven robust against modeling imperfections, exogenous disturbances (i.e., UDP traffic) and delays. Besides guaranteeing the saturated character of the proposed rate controller, an extra modification is also provided to further guarantee rate reduction whenever congestion detected.

This paper is organized as follows. Section II presents the control problem and preliminaries. Sections III and IV, analyze the proposed neural network rate control algorithm and the group sending time estimation algorithm. The illustrative simulation studies are performed in Section V. Finally, we conclude in Section VI.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Packet Switching Network System

We consider a general packet switching network. The topology of the network is characterized by a set of sources/receivers $C = \{1, 2, ..., n\}$, a set of nodes $Q = \{1, 2, ..., m\}$ and a set of links $L = \{1, 2, ..., l\}$ connecting the nodes. Each link $l \in L$ has an associated buffer with maximum capacity $B_l$. A source $S \in C$ has to transmit an application of prespecified amount of packets $N$ to a destination $D \in C$ through the network whose transmission rate $(u)$ is controlled by an appropriately designed protocol. Every source - receiver pair is characterized by a path $L(S, D)$, defined as the set of links the transmitted packets follow to reach their destination.

Upon arrival of a packet, the destination (receiver) issues an acknowledgment (ACK), which is received by the source. The amount of time that elapses between the instant the source starts to transmit a packet and the instant at which
it receives its ACK, is called Round Trip \((RT)\). In case of a packet loss (e.g., owing to a buffer overflow inside the network), no ACK will be received. The source waits for a certain amount of time called the timeout interval \((\tau_0)\), after which, if no ACK is received, the source proceeds to the packet retransmission.

Assuming for a moment that a single packet is transmitted every round trip time and no packet loss is present, the amount of time required for transmitting the application \((T_s)\) may be calculated on the basis of \(RT\) as \(T_s = \int_0^N RT(k)dk\).

The round trip time \((RT)\) of a source - receiver connection via a path \(L(S, D)\) is comprised of the transmission time \(T_t\) (defined as \(T_t = p_{size}/u, u \in [0, \pi]\) with \(p_{size}, \pi\) denoting the packet size and the maximum transmission rate respectively), the path propagation delay \(d_p\), as well as the buffering delay \(d_b\) formed in every non empty buffer in \(L(S, D)\). In other terms \(d_b = \sum_{j \in L(S, D)} d_b(j)\), where \(d_b(j)\) is the packet delay experienced in the \(j\)-th buffer.

Each link estimates a measure of its congestion level (denoted by \(p_{j}, j \in L(S, D)\)) which depends on the value of the incoming flow rate and the current buffer length. This information is transmitted through the path \(L(S, D)\) to the destination node, at which an aggregate path congestion measure \(p\) is formed via a nonlinear mechanism. In other terms \(p = F(p_1, p_2, ..., p_k), p \in [0, 1]\) with \(k\) denoting the number of links in \(L(S, D)\). Specifically, assuming \([7]\) that only the \(i\)-th link in a path is congested then \(p_i >> \sum_{j \neq i, j \in L(S, D)} p_j\) and hence with no loss of generality, \(p\) takes the form \(p = \sum_{j \in L(S, D)} p_k\).

In the literature, a variety of approaches have been adopted to reliably convey a coded path congestion information to the corresponding source. Indicatively, one-bit Explicit Congestion Notification (ECN) to code and transmit the congestion information \([9]\). In this work, whenever a packet arrives to be forwarded on a link, it is marked (ECN bit) with a probability whose value depends on the link buffer length. Hence, the congestion price is in the form of buffering delay.

**B. Problem Statement**

For a specific packet switching network system as described in the previous subsection, our primary concern is to design decentralized rate controllers, implemented at the source side, capable of guaranteeing high sensitivity to time quality characteristics, such as almost constant per packet delay, while preventing the path \(L(S, D)\) from congestion collapse and throughput from starvation.

The aforementioned design implies the existence of a control algorithm, capable of regulating the per packet \(RT\) close to a desired round trip time \(RT_{d}\), avoiding either overflow or empty link buffers. Let us assume for a moment that the source is about to transmit the \(k\)-th packet. Having knowledge of the path congestion level \(p\) this packet will experience in his travel through \(L(S, D)\), it is reasonable to claim that there exists a smooth, bounded but unknown nonlinear function of \(RT, u, p\) such as:

\[
\hat{RT} = f(\hat{RT}, u, p), \hat{RT} \in [0, \tau_0], u \in (0, \pi], p \in [0, 1] \quad (1)
\]

Besides the uncertainty in \(f()\), \((1)\) also requires knowledge of the future path congestion level \(p\), which is unfortunately unknown and thus has to be estimated. In addition, the a priori availability of feasible \(RT_{d}\) values is not a straightforward task, owing to the highly uncertain and dynamically varying character of the packet switching network. Hence, our control problem is re-formulated as follows:

**Problem 1.** Design: (i) a future path congestion level estimator, (ii) a feasible \(RT_{d}\) estimator, and (iii) a source rate controller capable of regulating \(e = RT - RT_{d}\) to an arbitrarily small neighborhood of zero, while keeping all other signals in the closed loop uniformly bounded.

The proposed transmission control framework in block diagram form is pictured in Fig. 1.

**C. Throughput Improvement**

In the proposed architecture, each source transmits one packet every \(RT\) and remains idle after transmission. However, the number of packets per source is high, which unavoidably leads to a significant increase in sending time, and throughput reduction. Apparently, improvement can be achieved by reducing the aforementioned idle time. To alleviate this problem, we introduce the notion of communication channels. Each source creates a number \((n)\) of communication channels that operate in parallel as if each one is a separate source. The general architecture of the rate control mechanism is presented in Fig. 2.

**III. TRANSMISSION CONTROL FRAMEWORK DESIGN**

**A. Future Path Congestion Level Estimator**

As already mentioned, each packet, when travelling through \(L(S, D)\), experiences an aggregate path congestion measure \(p(k), k = 1, 2, ..., N\) and every source requires...
knowledge of the future path congestion level $p(k+1)$. Under these conditions, a system for the estimation of the future path congestion level $p(k+1)$, should be implemented on every source. In this work, we consider a one-bit online assignment scheme for the estimation of the future path congestion level $p(k+1)$, which can be described as follows. Consider receiving the ACK of the $k$th packet. The incoming ECN bit $m(k)$ is either 0 or 1 provided the packet is marked or not respectively. Let $\hat{p}$ denote the estimate of $p$. The subsequent recursive formula (stable low-pass filter) is employed for the derivation of $\hat{p}(k+1)$:

$$\hat{p}(k+1) = \alpha \hat{p}(k) + (\alpha - 1)m(k), \quad k = 1, 2, \ldots, N$$  \hspace{1cm} (2)

where $\hat{p}(0) = 0$ denotes the initial condition on (2). The design constant $\alpha \in (0, 1)$ controls the significance of old estimates in deriving the new one. From (2) and the definition of $m(k)$ becomes apparent that $\hat{p} \in [0, 1]$, as it is requested, since the actual $p$ values also belong to $[0, 1]$.  

**Remark 1.** There are two sources of error in the above described estimation process. The first, which has the form of modeling error, is introduced when selecting (2) as a means to derive $\hat{p}$. Apparently, (2) constitutes the simplest possible implementation one can employ to obtain future path congestion estimated values. Certainly, other more complicated structures may be used to attenuate this source of error, raising however significantly the complexity. Furthermore, owing to significant delays (propagation and buffering) in the path $L(S, D)$, the estimates $\hat{p}$ as obtained from (2), actually correspond to past $p$ value. The latter constitutes the major cause of error in $\hat{p}$ derivation.

Owing to Remark 1 we may write:

$$p = \hat{p} + \bar{p}$$  \hspace{1cm} (3)

with $\bar{p}$ to denote the estimation error. The presence of a non-zero $\bar{p}$ affects the stability analysis of the closed loop system and will be considered in the subsections that follow.

**B. Neural Network Source Rate Controller Design**

In this section, we shall present a systematic tool, based upon a Lyapunov function derivative estimation approach, for the design of controllers capable to guarantee a uniform ultimate boundedness property for the tracking error $e = RT - RT_d$, as well as the uniform boundedness of all other signals in the closed loop.

Equation (1) can be expressed as $\dot{RT} = f_1(RT, u)f_2(\bar{p})$ where $f_1(RT, u)$ and $f_2(\bar{p})$, $\forall RT \in [0, \tau_0], \forall u \in (0, \pi]$ and $\forall \bar{p} \in [0, 1]$, are unknown, smooth, and bounded functions.  

Employing (3), $f_2(\bar{p})$ becomes

$$f_2(\bar{p}) = f_2(\hat{p}) + f_2(\bar{p}, \hat{p})$$  \hspace{1cm} (4)

where $f_2(\hat{p})$, $f_2(\bar{p}, \hat{p})$ are unknown, smooth, and bounded functions, with $f_2(\bar{p}, \hat{p})$ having the property $\lim_{\bar{p} \to \infty} f_2(\bar{p}, \hat{p}) = 0$. In the special case where $\bar{p} = 0$, $f_2(\bar{p}) \equiv f_2(\hat{p})$.

Substituting (4) into $\dot{RT}$ we obtain:

$$\dot{RT} = f_1(RT, u)f_2(\hat{p}) + f_1(RT, u)f_2(\bar{p}, \hat{p})$$  \hspace{1cm} (5)

Notice that $\bar{p}$ is not available for measurement and thus cannot be used for control. The following assumptions are necessary in the subsequent analysis.

**Assumption 1** There exists some unknown K-functions $g_i$, $i = 1, 2, 3$ such that $f_1(RT, u) \leq g_1(RT, u)$, $\forall RT \in [0, \tau_0], \forall u \in [0, \pi]$ and $f_2(\hat{p}, \bar{p}) \leq g_2(\bar{p})g_3(\bar{p})$, $\forall \bar{p} \in [0, 1]$.  

Moreover, since $g_3$ is a K-function there exists an unknown positive constant $d$ such that $g_3(\bar{p}) \leq d$.

**Assumption 2.** The solution of (5) can be forced to be uniformly ultimately bounded with respect to an arbitrarily small neighborhood of $e = 0$.

Owing to Assumption 2, there exists a radially unbounded robust control Lyapunov function $V(e) : \mathbb{R} \to \mathbb{R}_+$ satisfying $\lambda_1|e|^2 \leq V(e) \leq \lambda_2|e|^2$, $\lambda_1, \lambda_2 > 0$ and a control input $u$ such that $V = \frac{\partial V(e)}{\partial e}f_1(RT, u)f_2(\hat{p}) + f_1(RT, u)f_2(\bar{p}, \hat{p}) \leq 0 \forall RT, u, \bar{p}, \hat{p} \in \mathcal{A}$, where the set $\mathcal{A}$ is defined as $\mathcal{A} = \{RT, u, \bar{p}, \hat{p} \in \mathbb{R} : 0 \leq RT \leq \tau_0, 0 \leq u \leq \pi, \bar{p} < 0 < \hat{p} \}$ with $u_0$ an arbitrarily small positive constant. To proceed, let us define $A(e, RT, u, \bar{p}) = \frac{\partial V(e)}{\partial e}f_1(RT, u)f_2(\hat{p})$ and $B(e, RT, u, \bar{p}) = \frac{\partial V(e)}{\partial e}g_1(RT, u)g_2(\bar{p})$.

Since $f_1(RT, u), f_2(\hat{p}), g_1(RT, u), g_2(\bar{p})$, and $V(e)$ are assumed unknown, we may utilize the neural nets [7] and substitute the highly uncertain terms in $A, B$ by linear in the weights neural networks, plus a modeling error term $\forall e, RT, u, \bar{p} \in \Omega \subset \mathbb{R}$ where $\Omega = \{e, RT, u, \bar{p} \in \mathbb{R} : 0 \leq RT \leq \tau_0, 0 \leq u \leq \pi, \bar{p} < 0 < \hat{p} \}$. In other terms, there exist constant but unknown weight values $W^1_T, W^2_T$ such that $A(e, RT, u, \bar{p}) = W^1_T S_1(e, RT, u, \bar{p}) + \omega_1(e, RT, u, \bar{p})$ and $B(e, RT, u, \bar{p}) = W^2_T S_2(e, RT, u, \bar{p}) + \omega_2(e, RT, u, \bar{p})$. The following assumption is reasonable owing to the approximation capabilities of the linear in the weights neural nets.

**Assumption 3.** In a compact region $\Omega \subset \mathbb{R}^4$, $|\omega_1(e, RT, u, \bar{p})| \leq \delta_1$, $|\omega_2(e, RT, u, \bar{p})| \leq \delta_2$ where $\delta_1, \delta_2 \geq 0$ are unknown but small bounds.

Furthermore, the results obtained in this paper are semi-global in the sense that they are valid as long as $e, RT, u, \bar{p}$ remains in $\Omega$, where the set $\Omega$ can be made arbitrarily large. If Assumption 3 holds for all $e, RT, u, \bar{p} \in \Xi$, then the results becomes global.

Let us now consider the Lyapunov function candidate $L = kV(e) + \frac{1}{2}W^1_T [\hat{p}]^2 + \frac{1}{2}W^2_T [\bar{p}]^2 + \frac{1}{2}u^2$ where $V(e)$ is the previously defined robust control Lyapunov function. The parameter errors $\hat{W}_i$, $i = 1, 2$ are defined as $\hat{W}_i = W - W^*$, $i = 1, 2$. Differentiating with respect to time along the solutions of (5) we obtain $\dot{L} = kV(e) + \frac{1}{2}W^1_T \hat{W}_1 + \frac{1}{2}W^2_T \hat{W}_2 + u\hat{u}$.

Employing (5) and using Assumption 1 and $g_3(\bar{p}) \leq d$, $\hat{L}$ becomes $\dot{L} \leq kW^1_T S_1(e, RT, u, \bar{p}) + kW^2_T S_2(e, RT, u, \bar{p}) + k\omega(e, RT, u, \bar{p}) + W^1_T \hat{W}_1 + W^2_T \hat{W}_2 + u\hat{u}$.

For the generalized modeling error $\omega(e, RT, u, \bar{p})$ we may also assume that is uniformly bounded by a small yet unknown constant $\delta = \delta_1 + \delta_2$ $\forall e, RT, u, \bar{p} \in \Omega$.

Choosing $\hat{u} = \frac{1}{\gamma_1 - \gamma_2}(k - kW^1_T S_1(e, RT, u, \bar{p}) - kW^2_T S_2(e, RT, u, \bar{p}) - \gamma_1 e^2 - \gamma_2 \bar{p}^2)$, $\gamma_1, \gamma_2 > 0$
and

\[ W_i = -k_i W_i + k_S \epsilon(e, RT, u, \hat{p}, p_0), \quad k_i > 0, \quad i = 1, 2 \]  

(6)

Hence, \( \dot{L} \leq -\gamma_1 e^2 - \gamma_2 u^2 - \sum_{i=1}^2 \left[ \frac{k_i}{2} |W_i|^2 \right] + \mu \) where \( \mu = k \delta + \sum_{i=1}^2 \left[ \frac{k_i}{2} |W_i|^2 \right] \).

We assume that we start with an initial condition \( u(0) \in (u_0 \leq u \leq \pi) \) and we distinguish two cases.

**Case 1** \((u = \pi):\) The rate controller has reached its upper limit. Now \( \dot{u} \) is modified to \( \dot{u} = \frac{1}{\pi} k W_1^T S_1(e, RT, u, \hat{p}) - k W_2^T S_2(e, RT, u, \hat{p}) - \gamma_1 e^2 - \gamma_2 u^2 + \psi \) where

\[ \psi = \begin{cases} -M, & \text{if } u = \pi \text{ and } M - \gamma_1 e^2 - \gamma_2 u^2 > 0 \\ 0, & \text{otherwise} \end{cases} \]  

(8)

with \( M = k W_1^T S_1(e, RT, u, \hat{p}) - k W_2^T S_2(e, RT, u, \hat{p}) \).

Employing \( \dot{u} \), (8) it is straightforward to conclude that whenever \( u \) has reached its upper limit and has a tendency to move upwards (i.e., \( u = \pi \) and \( \dot{u} > 0 \)), then \( \dot{u} = \frac{1}{\pi} \left[ -\gamma_1 e^2 - \gamma_2 u^2 \right] < 0 \) guaranteeing \( u \leq \pi \) for all transmitted packets. Furthermore, \( \dot{u} \), (8) do not alter the stability properties proven thus far, since \( \dot{L} \) is augmented with the extra term \( \psi = k W_1^T S_1(e, RT, u, \hat{p}) + k W_2^T S_2(e, RT, u, \hat{p}) < -\gamma_1 e^2 - \gamma_2 u^2 < 0 \).

**Case 2** \((u = u_0):\) The rate controller has reached its lower limit. At this point and whenever there is a tendency to move downwards we substitute the terms \( S_1(e, RT, u, \hat{p}), \) \( i = 1, 2 \) by \( S_1(e, RT, u, \hat{p}) = S_1(e, RT, u, \hat{p}) b_i(W_i, e, RT, u, \hat{p}) + b_o(W_i, e, RT, u, \hat{p}) \), where

\[ b_i(W_i, e, RT, u, \hat{p}) = \begin{cases} r \frac{W_i^T S_i(e, RT, u, \hat{p})}{\|W_i^T S_i(e, RT, u, \hat{p})\|^2}, & \text{if } A_1 \\ 1, & \text{otherwise} \end{cases} \]  

(9)

\[ b_o(W_i, e, RT, u, \hat{p}) = \begin{cases} r \frac{W_i}{\|W_i\|^2}, & \text{if } A_2 \\ 0, & \text{otherwise} \end{cases} \]  

(10)

where \( r = \frac{3(\gamma_1 e^2 + \gamma_2 u^2)}{4k} \), \( A_1 = (u = u_0) \) and \( M - \gamma_1 e^2 - \gamma_2 u^2 < 0 \) and \( W_i^T S_i(e, RT, u, \hat{p}) \neq 0 \), \( A_2 = (u = u_0) \) and \( M - \gamma_1 e^2 - \gamma_2 u^2 < 0 \) and \( W_i^T S_i(e, RT, u, \hat{p}) = 0 \) for \( i = 1, 2 \).

Employing (9), (10), \( \dot{u} \) may straightforwardly conclude that whenever \( u = u_0 \) and \( M - \gamma_1 e^2 - \gamma_2 u^2 < 0 \) then \( \dot{u} = \gamma e^2 + \gamma u^2 > 0 \). However, (10) is valid, only when \( |W_i| \neq 0 \), \( i = 1, 2 \) which can be satisfied by applying the standard projection modification \( [S] \) on \( W_i, i = 1, 2 \).

**Reducing Rate in Congestion**

Whenever the source detects congestion in its path (\( \hat{p} = 1 \)), the only acceptable policy would be to reduce its rate until \( \ddot{p} < 1 \). To guarantee that \( \dot{u} \leq 0 \) whenever \( \ddot{p} = 1 \) (with \( u = 0 \) only when the rate controller has reached its lower limit \( u = u_0 \)), we modify \( \dot{u} \) as \( \dot{u} = \frac{1}{\pi} \left[ -k W_1^T S_1(e, RT, u, \hat{p}) - k W_2^T S_2(e, RT, u, \hat{p}) - \gamma_1 e^2 - \gamma_2 u^2 + \psi + \varphi \right] \) where

\[ \varphi = \begin{cases} -M, & \text{if } B_1 \\ -\frac{\gamma}{2} (\gamma_1 e^2 + \gamma_2 u^2), & \text{if } B_2 \\ -M + \gamma_1 e^2 + \gamma_2 u^2, & \text{if } B_3 \\ 0, & \text{otherwise} \end{cases} \]  

(11)

where \( B_1 = \ddot{p} = 1, u_0 < u < \pi \) and \( M - \gamma_1 e^2 - \gamma_2 u^2 > 0 \), \( B_2 = \ddot{p} = 1, u = u_0 \) and \( M - \gamma_1 e^2 - \gamma_2 u^2 < 0 \) and \( B_3 = \ddot{p} = 1, u = u_0 \) and \( M - \gamma_1 e^2 - \gamma_2 u^2 > 0 \).

To understand the operation of the proposed modification, we distinguish three cases.
Case 1 \((u_0 < u < \pi)\) and \(M - \gamma_1 e^2 - \gamma_2 u^2 > 0\): In this case, \(u \in (u_0, \pi)\) when \(\bar{p} = 1\) and at the same time \(u\) has a tendency to move upwards. Notice that \(\psi = 0\) in this case. Employing (11) into \(\hat{u}\), we obtain \(\hat{u} = -2a^2e^2 + w^2 < 0\).

Case 2 \((u = u_0)\) and \(M - \gamma_1 e^2 - \gamma_2 u^2 < 0\): Employing (9), (10), (11), \(\hat{u}\) we may straightforwardly conclude that \(\hat{u} = 0\), since \(\psi = 0\) for this case also.

Case 3 \((u = u_0)\) and \(M - \gamma_1 e^2 - \gamma_2 u^2 > 0\): Employing (11), \(\hat{u}\) we may straightforwardly conclude that \(\hat{u} = 0\).

Furthermore, since in each case, we add the negative terms \(-M, -\frac{1}{2}(\gamma_1 e^2 + \gamma_2 u^2)\) and \(-M + \gamma_1 e^2 + \gamma_2 u^2\) respectively in \(\hat{L}\), the stability properties proven thus far are not harmed.

IV. GROUP SENDING TIME ESTIMATION ALGORITHM

In the proposed transmission control framework, each source has to transmit an application of \(N\) packets to a destination through the network. The adaptive control formulation provided requires knowledge of a desired RT per packet value (\(RT_d\)) for all application packets. Unfortunately, no such information is available to the source a priori. To establish a feasible solution, an estimated sending time mechanism should be constructed at the source, aiming at providing reliable reference to the rate controller.

Assume for an instant that a single packet is transmitted every round trip and let \(T_s\) denote the estimated application sending time. The corresponding estimated round trip time \(RT_d\) can be derived as \(RT_d = T_s/N\).

In the presence of communication channels, the total number of packets is divided into groups of \(M\) packets and the channels undertake the parallel transmission of each packet in the group. However, now the target round trip time per channel value is provided by \(RT_t = \frac{\hat{M}^2}{M}\) where \(\hat{M}\) denotes the estimated group sending time and \(\eta\) the number of communication channels.

Let us assume that we have already transmitted \(k\) groups and we are about to transmit the \(k+1\) group. Let \(y_a(k+1)\) and \(y_a(k)\) be the achieved sending times for \(k + 1\) and \(k\) groups respectively while \(\hat{p}(k)\) denotes the path congestion level predicted at the beginning of the \(k\)-th group transmission. Assuming for the moment almost constant \(\hat{p}\) during the transmission of the \(k\)-th group, we argue that the variation \(\Delta y_a(k) = y_a(k+1) - y_a(k)\) depends on the number of channels (taken constant), as well as past path congestion levels and group sending times. Hence, \(\Delta y_a(k)\) can be described by:

\[
\Delta y_a(k) = \hat{P}(D(k)), \quad y_a(0) = 0
\]

where \(\hat{P}(D(k))\) is an unknown, sufficiently smooth and bounded function of \(D(k) = \{y_a(k), y_a(k-1), y_a(k-2), \ldots, y_a(k-m_1), \hat{p}(k), \hat{p}(k-1), \hat{p}(k-2), \ldots, \hat{p}(k-m_2)\}\) with properly selected positive values of \(m_1, m_2\).

Apparently, owing to the dynamic character of the Internet, the almost constant \(\hat{p}\) value assumption during group transmission is highly unrealistic, especially when the number of packets in a group \(M\) is significant. To alleviate this problem we propose the constant monitoring of the \(\hat{p}\) per packet value in a group and whenever a norm of the difference between the current \(\hat{p}\) value and the one predicted at the beginning of the group transmission exceeds a certain predetermined threshold \(dp\), then the group transmission is terminated and the procedure is repeated by sending the next \(M\) packets with a new \(y_a\) estimate.

To continue, we can assume, with no loss of generality, that there exists weight values \(W_a^* \in \mathbb{R}^L\) and an appropriately defined regressor vector \(S_a \in \mathbb{R}^L\), such that the unknown system (12) can be completely described by

\[
y_a(k+1) = y_a(k) + W_a^T S_a(D(k)) + \omega_a(D(k)) \tag{13}
\]

For the modeling error term \(\omega_a(D(k))\) we have:

Assumption 4. There exist an arbitrarily small positive constant \(\delta_a\) such that \(|\omega_a(D(k))| \leq \delta_a, \quad \forall y_a \in \Omega_a\) and \(\forall p(k) \in [0, 1]\), where \(\Omega_a\) is a compact region.

Defining \(\hat{W}_a(k)\) as the estimate of \(W_a^*\) at group \(k\), we generate the estimated value \(\hat{y}_a(k+1)\) of the group sending time \(y_a(k+1)\) as

\[
\hat{y}_a(k+1) = \hat{y}_a(k) + \hat{W}_a^T(k) S_a(D(k)) + \beta e_a(k) \tag{14}
\]

where \(\beta \neq 0\) and \(e_a(k)\) is the sending time estimation error defined as \(e_a(k) = y_a(k) - \hat{y}_a(k)\).

From (13), (14) we obtain \(e_a(k+1) = (1 - \beta) e_a(k) - (\hat{W}_a^T(k) S_a(k)) + \omega_a(D(k))\) where \(\hat{W}_a(k) = \hat{W}_a(k) - W_a^*\) the weight estimation error and \(\beta \neq 0, 1\).

Let us take the Lyapunov function candidate \(J = e_a^2(k) + \hat{W}_a^T(k) \hat{W}_a(k)\). Its first difference is given by \(\Delta J = e_a^2(k + 1) - e_a^2(k) + \hat{W}_a^T(k + 1) \hat{W}_a(k + 1) - \hat{W}_a^T(k) \hat{W}_a(k)\).

Take the weight update law as

\[
\hat{W}_a(k+1) = \mathcal{P}_a\{\hat{W}_a(k) + (1 - \beta) e_a(k) S_a(D(k))\} \tag{15}
\]

where \(\beta \neq 0, 1\) and \(\mathcal{P}_a\) denotes the projection operator with respect to the convex and bounded set \(\mathcal{R}_a\), which is defined as \(\mathcal{R}_a = \{W_a \in \mathbb{R}^L : \|W_a\| \leq w_m, \quad w_m > 0\}\).

Since \(S_a(D(k))\) and \(W_a^*\) are bounded by construction, we have \(|S_a(D(k))| \leq S_m\) and \(|\hat{W}_a(k)\| \leq 2w_m\).

Substituting (14), \(e_a(k+1)\) and (15) in \(\Delta J\) finally obtain

\[
\Delta J \leq \beta |e_a(k)|^2 + 2(1 - \beta)(\delta_a + w_m S_m)|e_a(k)| + 4|\hat{W}_a^T(k) S_m^2 + \beta^2 + 2\delta_a w_m S_m\) where \(\beta = (1 + S_m^2)(\beta - 2(1 + S_m^2)) + S_m^2\) with \(\beta < 0\) whenever \(\beta \in (1 - \sqrt{1/(1 + S_m^2)}, 1) \cup (1, 1 + \sqrt{1/(1 + S_m^2)})\)

Hence, \(\Delta J \leq 0\) provided that \(|e_a(k)| \geq -1 - \beta|\delta_a + w_m S_m)/\beta + \sqrt{(\delta_a + w_m S_m)^2[1 - \beta^2]/\beta^2 - \beta \bar{b}/\beta\}, \quad \bar{b} = \|W_a^T(k) S_m^2 + \beta^2 + 2\delta_a w_m S_m\|\). Therefore, \(e_a(k)\) possesses a uniform ultimate boundedness property with respect to the set \(\mathcal{E}_a = \{e_a(k) : |e_a(k)| \leq -1 - \beta|\delta_a + w_m S_m)/\beta + \sqrt{(\delta_a + w_m S_m)^2[1 - \beta^2]/\beta^2 - \beta \bar{b}/\beta}\} \). Notice that \(\delta_a\) is an arbitrarily small positive constant.

Further, selecting \(\beta\) close to unity, then \(\beta \simeq -1\) and \(\mathcal{E}_a\) becomes \(O(W_a^\top)\). Unfortunately, choosing \(\beta \simeq 1\) has the negative effect of slowing down the adaptation of \(\hat{W}_a\) given by (15). Obviously, a compromise between the size of \(\mathcal{E}_a\) and the adaptability of \(\hat{W}_a\) is necessary. We have thus proven the theorem:

Theorem 2. Consider the system (13). The group sending time estimation algorithm (14), (15) with \(\beta \in \{1 -
background traffic. All routers mark incoming packets with random CBR source is persistently submitting UDP packets behaviors, as well as the estimated future path congestion the corresponding round trip per packet error is presented Fig. 4(b). We selected the first channel of the first source Fig. 4(a). The queue length variations of the second router transmit 25000 packets.

3URFHHGLQJVRIWKHWK0HGLWHUUDQHDQ&RQIHUHQFHRQ \( S_i - D_i \) and one random Constant Bit Rate (CBR) source. The first consists of \( S_i - D_i, i = 1, 2, 3, 4 \) with double propagation delay than the second \( S_i - D_i, i = 5, 6, 7, 8 \). The random CBR source is persistently submitting UDP packets with average rate of 10 Mbps, thus consisting the network background traffic. All routers mark incoming packets with probability \( p_m \) equal to \( \frac{\text{buffer level}}{100} \), while the number of communication channels for every source is equal to 10. Each NNRC \( (S_i, i = 1, 2, 3, 4) \) source has to transmit 15000 packets, while each NNRC \( (S_i, i = 4, 5, 6, 7) \) source has to transmit 25000 packets.

The sending times acieved by the proposed congestion control algorithm appear in Table 1. During the simulation the bottleneck utilization always stays at 100% as seen in Fig. 4(a). The queue length variations of the second router (R-2) with respect to its maximum capacity are presented in Fig. 4(b). We selected the first channel of the first source and we plot the achieved RT per packet in Fig 4(c), while the corresponding round trip per packet error is presented in Fig 4(d). The transmission rate required to achieve this behaviors, as well as the estimated future path congestion level \( \beta \) are shown in Figs 4(e) and 4(f) respectively. Both the estimated and achieved group sending times are plotted in Fig 4(g). The almost constant transportation delay behavior may be observed from Fig 4(c). Finally in Fig 4(h) we show the throughout of source 1. Similar behavior can be shown to hold for the rest of the channel - sources of the network.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Utilization</th>
<th>Queue Length (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>0,5msec</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0,5msec</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0,5msec</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0,5msec</td>
</tr>
</tbody>
</table>

\( \sqrt{1/(1 + S_{im}^2)} \bigcup (1, 1 + \sqrt{1/(1 + S_{im}^2)}) \). \( |S_u(D(k))| \leq S_m \) guarantees the uniform ultimate boundedness of the group sending time estimation error \( e_a(k) = \hat{y}_a(k) - \hat{y}_a(k) \), with respect to the set \( \mathcal{E}_a \), which can be made \( O(w_m) \) by appropriately selecting \( \beta \).

VI. CONCLUSIONS

A stable decentralized neuro-adaptive congestion control scheme is presented in this paper capable of regulating the per packet round trip, thus achieving almost constant delay. Robustness against modelling imperfections, exogenous disturbances and delays has also been proven. To improve throughput, the notion of communication channels has been introduced. The proposed controller is guaranteed to be saturated. Modifications are also provided to achieve rate reduction whenever congestion is detected. Simulation studies have been performed to highlight the performance of proposed control scheme.

REFERENCES