A Prony-like polynomial-based approach to model order reduction

L. Coluccio, A. Eisinberg, G. Fedele
Dip. Elettronica Informatica e Sistemistica, Università degli Studi della Calabria, 87036 Rende (CS), Italy

Abstract—Presented in this paper is a Prony-like polynomial-based method to identify reduced-order linear models using response-matching technique. Impulse and step inputs have been considered. The poles configuration is automatically chosen by the solution of a new regression equation which uses modified signals obtained directly from the responses. The approach uses the algebraic derivatives method in the frequency domain yielding exact formula in terms of multiple integrals of the signal, when placed in the time domain. The effectiveness of the proposed method is shown by simulated experiments.

I. INTRODUCTION

In many real cases a complex high order model of the system under consideration is obtained from theoretical considerations. This complexity often makes difficult to understand the system behavior or to design controllers for such systems. A popular approach to make easier this situation is model reduction, where an approximate small order model is used in place of the original system.

A consistent number of model-reduction methods are available in literature. These methods can essentially be grouped into two families: the first utilizes the projection of the original high-dimensional dynamical system onto a suitable low-dimensional subspace [1], [2], [3]. This approach originated mainly in the field of structural dynamics, often uses the concept of model reduction. The second approach has mainly been developed in the control community. Here one tries to approximate the transfer function of the dynamical system in a suitable way. The approximation can be performed in the frequency-domain or in the time-domain where the methods using response error minimization occupy a premier position among them [4], [5], [6], [7]. A broad classification of order-reduction methods using response-matching techniques can be made in different ways like the one according to the type of input, i.e., sinusoidal signal or step and impulse input [8]. Order reduction of linear systems using step and impulse response matching involves the error minimization problem which has been handled with classical techniques such as gradient-based search and optimization approaches yielding some problems:

- dependency on the initial point chosen for the convergence to an optimal solution;
- global solution not necessarily found out;
- an algorithm efficient in solving one search and optimization problem may not be efficient in solving a different problem;

To overcome such problems, a genetic algorithm-based method is proposed in [9].

Presented in this paper is a method for obtaining reduced-order models using a modified Prony method [10]. Conventional Prony analysis is a signal analysis technique that fits a signal to a weighted summation of \( n_p \) damped modal components in the form

\[
y(t) = \sum_{k=1}^{n_p} R_k e^{p_k t}
\]

with pairwise unknown parameters \( \{R_i, p_i\}_{i=1}^{n_p} \), which are to be found from a given discrete sequence of noisy data \( \{y(k)\}_{k=1}^{n} \) obtained from some experiment. It is important to note that the conventional Prony analysis identifies a model of a signal, not a system or transfer function. The idea of using a Prony method for model order reduction is not new. In [11], [12] a method based on Prony analysis to obtain transfer functions for power-system-stabilizer is discussed. The method presented in this paper is close to the approach in [13] where an extension of the Prony method to non-uniform sampling is discussed. The technique is based on two steps. First, the response of the original system to impulse or unit step input, is expanded into a finite series of shifted Gram polynomials [14]. It is well known that some simple pre-treatment of the step response data can be used to enhance greatly the accuracy of the model [15]. Second, we propose to apply a Prony-like method to reduce the order of the system. The remainder of the paper is organized as follows. In Section II, the Prony-like method is reviewed. Section III contains some useful properties about the estimation of the Gram model from available data. In Section IV the proposed model-order reduction is discussed and simulations of high-order systems are presented. Finally, Section V provides some concluding remarks.

II. A PRONY-LIKE METHOD

Let

\[
Y(s) = \frac{R_1}{s-p_1} + \frac{R_2}{s-p_2} + \ldots + \frac{R_{n_p}}{s-p_{n_p}}
\]

be the Laplace transform of \( y(t) \). Define the polynomial

\[
B(s) = \prod_{k=1}^{n_p} (s-p_k)
\]

of degree \( n_p \), so that
\[
B(s) = \sum_{i=0}^{n_p} \sigma(n_p, n_p - i)s^i,
\]
where
\[
\sigma(n_p, k) = (-1)^k \sum_{i=1}^{k} p_{n_p}p_{n_p-i}...p_{n_p-k}
\]
is the \(k\)th order elementary symmetric function associated with \(\{p_1, p_2, ..., p_{n_p}\}\).

The following result is obtained [13].

**Theorem 1:**
\[
\sum_{i=0}^{n_p} \sum_{j=1}^{n_p} \left( \begin{array}{c} n_p \\ n_p-j \end{array} \right) \left( \begin{array}{c} n_p-i \\ n_p-j \end{array} \right) (n_p-j)!s^{j-i} \frac{d^jY(s)}{ds^j} \sigma(n_p, i) = 0.
\]

To avoid the derivative operation in the time domain, the division by \(s^{n_p+1}\) is applied to (6):
\[
\sum_{i=0}^{n_p} \sum_{j=1}^{n_p} \left( \begin{array}{c} n_p \\ n_p-j \end{array} \right) \left( \begin{array}{c} n_p-i \\ n_p-j \end{array} \right) (n_p-j)!s^{j-i} \frac{d^jY(s)}{ds^j} \sigma(n_p, i) = 0.
\]

Taking the inverse Laplace transform, (7) can be expressed in time domain as:
\[
\sum_{i=0}^{n_p} \beta(i, t) \sigma(n_p, i) = 0,
\]
where
\[
\beta(i, t) = \sum_{j=1}^{n_p} \left( \begin{array}{c} n_p \\ n_p-j \end{array} \right) \left( \begin{array}{c} n_p-i \\ n_p-j \end{array} \right) (n_p-j)! \int \left[ j^{(n_p+1+i-j)} (-1)^i t^i y(t) \right],
\]
with the integral expression
\[
\int_0^t \int_0^{x_1} ... \int_0^{x_{j-1}} \phi(x_j)dx_j...dx_1,
\]
with the definition
\[
\int_0^1 \phi(t) = \int_0^t \phi(x_1)dx_1.
\]

**Remark 1:** As in all Prony-like methods the problem to estimate the unknown parameters has been expressed as a separable regression. While Prony’s method and its modified version are performed in the \(z\)-domain [16], the novel approach performs in the \(s\)-domain allowing us to collect the data values also by non uniform sampling.

**Remark 2:** While in Prony’s method the roots of the regression equation are expressed in terms of the elementary symmetric functions on \(p_{k}T_s\), \(k = 1, 2, ..., n_p\), where \(T_s\) is the sampling period of the collected data, in the proposed method the roots of (8) contain the elementary symmetric functions on \(p_k, k = 1, 2, ..., n_p\).

To avoid the numerical integrations in (9), it can be used a polynomial \(p(t)\) of degree \(m - 1\) which fits \(y(t)\) in the least-squares sense:
\[
p(t) = \sum_{k=1}^{m} \hat{a}_k t^{k-1}.
\]

By using the following facts:
\[
\int (n_p+i-j) t^{k+j-1} = \frac{t^{n_p+k+i} (k+j)!}{(n_p+k+i)!},
\]
\[
\sum_{j=0}^{n_p} (-1)^j \left( \begin{array}{c} n_p \\ n_p-j \end{array} \right) (n_p-j)! (k+j-1)!
\]
\[
= \frac{\Gamma(n_p+1-i-k) \Gamma(k)}{\Gamma(1-i-k)}
\]
and standard properties of the gamma function [17], we obtain:
\[
\beta(i, t) = (-1)^n_p \sum_{k=n_p+1-i}^{m} \frac{\Gamma(i+k) \Gamma(k) \hat{a}_k t^{n_p+k+i}}{\Gamma(1+k-n_p) \Gamma(1-i-k)}.
\]

Let \(\{t_i, i = 1, 2, ..., n\}\) be a set of time instants where \(\beta(i, t)\) will be evaluated. Equation (8) can be expressed in matrix form as follows:
\[
VM \sigma = Vb
\]
where \(V \in \mathbb{R}^{n \times m}, M \in \mathbb{R}^{m \times n_p}, b \in \mathbb{R}^{n}\) and \(\sigma = \{\sigma(n_p, n_p+1-i), i = 1, 2, ..., n_p\}\) with
\[
V(i, j) = t_i^{2n_p+j}, \quad i = 1, 2, ..., n; \quad j = 1, 2, ..., m,
\]
\[
M(i, j) = \begin{cases} (-1)^n_p \frac{\Gamma(n_p+i+j) \Gamma(i+j-1) \Gamma(i+j)}{\Gamma(i) \Gamma(2n_p+i+j)} & j = 1, 2, ..., n_p, i = 1, 2, ..., m + 1 - j, \\ 0 & \text{otherwise,} \end{cases}
\]
\[
b(i) = \begin{cases} (-1)^n_p \frac{\Gamma(n_p+i+j)^2 \Gamma(i+j) \Gamma(2n_p+i+j)}{\Gamma(i) \Gamma(2n_p+i+j)} & i = 1, 2, ..., m - n_p, \\ 0 & \text{otherwise.} \end{cases}
\]

An estimation \(\hat{\sigma}\) in the least-squares sense can be found as
\[
\hat{\sigma} = (MTV^TVM)^{-1} M^TVb.
\]

Parameters \(R_i, i = 1, 2, ..., n_p\) are computed following the same line of the Prony’s method [10].

**III. GRAM POLYNOMIALS**

Let \(Z_n = \{z_r | z_r = z + rT_s, r = 0, 1, ..., n-1\}\) be a set of \(n\) equidistant points with sampling period equal to \(T_s\) and introduce the discrete inner product:
\[
(f_1(t), f_2(t)) = \sum_{r=0}^{n-1} f_1(z_r)f_2(z_r).
\]
A family \( P = \{p_1(t), p_2(t), \ldots, p_m(t)\} \) with \( m \leq n \) of polynomials is orthogonal respect to this inner product if the following properties hold:

\[
(p_k(t), p_q(t)) = 0, \quad k \neq q; \quad k, q = 1, 2, \ldots, m \\
(p_k(t), p_k(t)) = p_k \neq 0, \quad k = 1, 2, \ldots, m.
\] (18)

If \( p_k = 1, \quad k = 1, 2, \ldots, m \) then polynomials \( p_k \) are said orthonormal on the set of nodes \( Z_n \). Orthogonal polynomials on the set \( Z_n \) have the following explicit expression [14]:

\[
p_k(t) = \sum_{s=1}^{k} (-1)^s \binom{k}{s} \binom{n-s}{s-1} \binom{t_n}{s-1}.
\] (19)

This family satisfies the three-term recurrence relation

\[
p_k(t) = (\alpha_k t + \beta_k) p_{k-1}(t) + \gamma_k p_{k-2}(t), \quad k = 3, 4, \ldots, m
\] (20)

where

\[
\begin{align*}
\alpha_k &= \frac{4k-6}{T_s(k-1)^2}, \\
\beta_k &= -\frac{(2k-3)(2s+T_s(n-1))}{T_s(k-1)^2}, \\
\gamma_k &= \frac{(k-2)^2-n^2}{(k-1)^2}, \quad k = 3, 4, \ldots, m
\end{align*}
\] (21)

and the discrete inner product

\[
\rho_k = \left( \frac{n+k-1}{2k-1} \right) \left( \frac{2k-2}{k-1} \right), \quad i = 1, 2, \ldots, m. \] (22)

Here we want to expand a given function \( y = f(t) \) in terms of orthogonal polynomials:

\[
f(t) = \sum_{i=1}^{m} w_i p_i(t) \] (23)

and to determine the coefficients \( w_1, w_2, \ldots, w_m \) such that the Euclidean norm of the error function \( \tilde{f} = f \) is minimized,

\[
||\tilde{f} - f|| = \sum_{j=0}^{n-1} |f(z_j) - f(z_j)|^2.
\]

Since \( p_1(t), p_2(t), \ldots, p_m(t) \) form an orthogonal system, the coefficients are computed more simple by

\[
w_i = \frac{\langle p_i(t), f(t) \rangle}{\langle p_i(t), p_i(t) \rangle}, \quad i = 1, 2, \ldots, m.
\] (24)

If we define the following quantity:

\[
b = H \cdot f
\] (25)

where

\[
H(i, j) = p_i(z_j), \quad i = 1, 2, \ldots; \quad j = 0, 1, \ldots, n-1
\] (26)

\[
f = [f(z_0), f(z_1), \ldots, f(z_{n-1})]
\] (27)

then

\[
w_i = \frac{b(i)}{\rho_i}, \quad i = 1, 2, \ldots, m.
\] (28)

Since the product \( b = H \cdot f \) is invariant under affine transformation, then it would be computed by considering, for example, the set of orthogonal polynomials on \([1, 2, \ldots, n]\) for which the matrix \( H \) is

\[
H(i, j) = \sum_{s=1}^{i} (-1)^{s+i} \binom{s+i-2}{s-1} \binom{n-s}{n-i} \binom{j-1}{s-1}, \quad i = 1, 2, \ldots, n
\] (29)

Remark 3: Although high degree approximations on equidistant nodes have a bad reputation in numerical analysis, a lot of researches have been made for the design of accurate algorithms for polynomial approximation on this set of nodes, since this choice frequently occurs in many applications. Moreover it is known that the polynomials in the family \( \{P_i\}_{i=1}^{m} \) are of modest size on \([-1, 1]\) when \( m \leq 2.5m^{1/2} \), and they are therefore well suited for the approximation of functions on this interval [18].

IV. PROPOSED MODEL-ORDER REDUCTION

Assume that for a unit step input change, the output response of a stable, linear, time-invariant system with transfer function \( G(s) \) is given by \( y_u(t) \), possibly as a vector of data points. The problem is to obtain an estimate of the impulse response by differentiating the step response, i.e. \( h(t) = \frac{dy_u(t)}{dt} \), and then using this polynomial estimated model in place of the impulse response \( h(t) \). Here, the shifted-Gram polynomials are used as a type of modulating functions. The estimated \( (m-2) \)th-order shifted-Gram model from noisy-step response data is

\[
\hat{h}(t) = \sum_{i=1}^{m} c_i \frac{dp_i(t)}{dt} = \sum_{i=1}^{m-1} a_i t^{i-1},
\] (30)

where \( p_i(t) \) is the Gram polynomial of degree \( i-1 \) with \( z = 0 \). Let \( \tilde{c} = \{c_i, i = 1, 2, \ldots, m\} \) be the coefficients in (30), then coefficients \( a_i, i = 1, 2, \ldots, m-1, \) can be calculated as the first \( m-1 \) elements of the vector

\[
\tilde{a} = \tilde{c}^T C_P M_D,
\] (31)

where \( C_P \) is the lower triangular matrix of the coefficients of the polynomials \( p_i(t) \) which can be constructed by using the three-term recurrence relation (21) (if \( p_1(t) = \sum_{k=1}^{N} l_{k,i} t^{i-1} \) then \( C_P(i, j) = l_{i,j}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, i \) and

\[
M_D(i, j) = \begin{cases} 
1 - 1, & i = 2, 3, \ldots, m; \quad j = i - 1, \\
0, & \text{otherwise.}
\end{cases}
\]
Therefore a Prony-like method, as described in Section II, is applied to find the reduced model of order \( n_p \) less than the true system order. If the impulse response is available then \( h(t) \) can be directly approximated in terms of \( m \) Gram polynomials

\[
\hat{h}(t) = \sum_{i=1}^{m} c_i p_i(t) = \sum_{i=1}^{m} a_i t^{i-1}.
\]

where \( f(t) \) is the impulse response of the original system and \( g_{(mt)}(t) \) is the impulse response of the reduced system using method \( mt = \{ Lu, CEF \} \):

\[
ISE_{(Lu)} = 2.206 \cdot 10^{-3},
\]

\[
ISE_{(CEF_{step})} = 1.969 \cdot 10^{-3}.
\]

Analogous results are obtained by using an impulse as input.

1) Example 1: The system used in [19] is given as the fifth-order transfer function

\[
F(s) = \frac{156 + 369s + 264s^2 + 80s^3 + 10s^4}{40 + 148s + 173s^2 + 84s^3 + 21s^4 + 2s^5}.
\]

In [19] it is required to find the coefficients \( x_i \) in the second-order reduced model

\[
G_2(s) = \frac{3.9 + x_1 s}{1 + x_2 s + x_3 s^2}
\]

so that the deviations between the original system and the reduced system in the Nyquist diagrams are minimized over the frequency range of interest. The resulting optimal model using the global optimization method proposed by Luus and Jaakola in [20] is

\[
G_{2}^{Lu}(s) = \frac{3.9 + 3.77664s}{1 + 2.30008s + 0.78956s^2}
\]

By using our method, with a unit step input, we obtain

\[
G_2^{CEF_{step}}(s) = \frac{5.14986 + 4.8255s}{1.32213 + 2.98676s + s^2}
\]

with \( m = 20, n = 1000, T_s = 0.008 \).

To measure the quality of the reduced model, we use the performance index

\[
ISE_{(mt)} = \int_{0}^{\infty} (f(t) - g_{(mt)}(t))^2 dt
\]

A. Numerical experiments

We report here some numerical experiments to investigate the effectiveness of the proposed method. From simulation results and comparisons with previous works [19], [20], [21], we can recognize that it provides accurate simulation results and comparisons with previous works which use a different optimality criterion.

1) Example 2: In [19] the system of Krishnamurthy and Seshadri [21] is also considered, where the eighth-order transfer function have the poles

\[
\begin{align*}
    p_{1.2} &= -1 \pm j, \\
    p_3 &= -1, \\
    p_4 &= -3, \\
    p_5 &= -4, \\
    p_6 &= -5, \\
    p_7 &= -8, \\
    p_8 &= -10.
\end{align*}
\]

The zeroes of the transfer function are

\[
\begin{align*}
    z_{1.2} &= -1.0346 \pm 0.63102j, \\
    z_3 &= -2.6369, \\
    z_4 &= -3.6345, \\
    z_5 &= -4.9021, \\
    z_6 &= -7.8014, \\
    z_7 &= -9.7845.
\end{align*}
\]

The optimal second-order model in [19] is

\[
G_2^{Lu}(s) = \frac{20.2583 + 11.62826s}{1 + 1.06333s + 0.35299s^2}
\]

with \( ISE_{(Lu)} = 3.197 \cdot 10^{-1} \).

To show the effectiveness of our method, we test it on the impulse response, with \( m = 18, T_s = 0.0008 \) and \( n = 5000 \). The estimated second order model is

\[
G_2^{CEF_{impulse}}(s) = \frac{42.0953 + 33.2799s}{2.12075 + 2.00678s + s^2}
\]

with \( ISE_{(CEF_{impulse})} = 2.361 \cdot 10^{-1} \). Figure 1 compares impulse responses of \( G_2^{Lu}(s) \) and \( G_2^{CEF_{impulse}}(s) \).

![Fig. 1. Example 2: Comparison between impulse response of our estimated 2nd order model and Luus and Jaakola one.](image)

The optimal fourth-order model in [19] is
\[ G_{Lu}(s) = \frac{20.2585 + 32.4653s + 20.5435s^2 + 3.0402s^3}{1 + 2.1271 + 1.8271s + 0.1523s^2 + 0.9870s^3}, \] with \( ISE(M_{Lu}) = 2.306 \cdot 10^{-4}. \)

Our method, by impulse response matching, with \( m = 15, T_s = 0.001 \) and \( n = 5000 \), gives he following result

\[ G_{CEF_{impulse}}^4(s) = \frac{266.519 + 392.627s + 243.452s^2 + 34.9224s^3}{13.1074 + 26.235s + 21.9213s^2 + 8.8602s^3 + s^4}, \] with \( ISE(G_{CEF_{impulse}}) = 2.56 \cdot 10^{-4}. \) In case of step response we obtain

\[ G_{CEF_{step}}^4(s) = \frac{238.925 + 486.405s + 240.319s^2 + 34.0107s^3}{12.7914 + 29.738s + 21.0007s^2 + 8.7158s^3 + s^4}, \] with \( ISE(G_{CEF_{step}}) = 2.79 \cdot 10^{-4}, m = 18, T_s = 0.001 \) and \( n = 5000. \)

3) Example 3: The example given below was considered by Mukherjee et al. in [8]. The high-order system have four zeroes

\[ \begin{align*}
z_1 & = -3 + 2.8284j, \\
z_2 & = -3 - 2.8284j, \\
z_3 & = -4, \\
z_4 & = -25,
\end{align*} \]

and nine poles

\[ \begin{align*}
p_1 & = -1, \\
p_2 & = -1 + 2j, \\
p_4 & = -1 + 2j, \\
p_6 & = -1 + 3j, \\
p_9 & = -1 + 4j.
\end{align*} \]

In [8] the order-reduction method, having different pole configurations like real only, mixed real and complex pair, and repeated ones, is performed using step and impulse response-matching technique. In case of step input, matching is partially defined a priori i.e., only the steady-state parts of the step input response are exactly matched whereas the ISE between the transient parts of the same is minimized using the secant method. Since our method, performed by using a unit step input, gives a reduced model with one real and a pair of complex poles, we compare our estimation with Mukherjee et al. one, that is

\[ G_{Mu_{step}}^3(s) = \frac{3.035 - 1.18s + 0.8117s^2}{3.035 + 4.682s + 2.718s^2 + s^3}. \] (41)

By using our method we obtain

\[ G_{CEF_{step}}^3(s) = \frac{2.58504 - 1.28465s + 0.227092s^2}{2.54021 + 4.24996s + 2.4386s^2 + s^3}, \] with \( m = 15, n = 1000, T_s = 0.006. \) ISE indexes are

\[ \begin{align*}
ISE(M_{Mu_{step}}) & = 8.559 \cdot 10^{-2}, \\
ISE(CEF_{step}) & = 1.162 \cdot 10^{-2}.
\end{align*} \]

Comparison between impulse response of our estimated 3rd order model and Mukherjee et al. one is shown in Fig. 2. Moreover step responses of our estimated 3rd order model and Mukherjee et al. one are reported in Fig. 3. From the obtained results it is clear that our estimated model performs better than Mukherjee et al. one especially in the transient response. In case of impulse response matching, our method gives the following 3rd order model

\[ G_{CEF_{impulse}}^3(s) = \frac{2.58985 - 1.28711s + 0.229213s^2}{2.54442 + 4.25468s + 2.44076s^2 + s^3}, \] (46)

with \( ISE(G_{CEF_{impulse}}) = 1.624 \cdot 10^{-2}. \) The corresponding model in [8], with one real and a pair of complex poles is

\[ G_{Mu_{impulse}}^3(s) = \frac{2.329 - 2.202s + 0.2945s^2}{2.32887 + 4.76973s + 2.5081s^2 + s^3}, \] (47)

with \( ISE(G_{Mu_{impulse}}) = 7.492 \cdot 10^{-2}. \)

The estimated 4th order model, by using our method, is

\[ G_{CEF_{step}}^4(s) = \frac{11.10914 - 5.99943s + 1.10916s^2 - 0.109976s^3}{11.2843 + 1.845s + 12.7207s^2 + 4.40412s^3 + s^4}, \] (48)

with \( ISE(G_{CEF_{step}}) = 1.909 \cdot 10^{-3}. \)

V. CONCLUDING REMARKS AND FUTURE LINES SEARCH

In this paper method to reduce the order of a given model is presented. Central in this method is the adoption of a polynomial series expansion of the system output. The choice of polynomials used in the approximation of the output signal is reverted on shifted Gram polynomials, orthogonal on a set of equidistant points, since the uniform logic of collecting samples often occurs in many practical situations. Clearly other basis functions can be used, for example Laguerre ones [22]

\[ L_i(t, \alpha) = \frac{e^\alpha t^{-\alpha}}{\Gamma(\alpha)} \frac{d^i}{dt^i} (e^{-\alpha t^i + \alpha}). \]
Motivating the further study of Laguerre models is the observation that approximations on such models, typically, provide rapid convergence with consequent reduction in the number of terms needed for accurate approximation. Moreover it can be shown that multiple integrals of \( y(t) \) (expressed as a truncated series of Laguerre functions \( L_i(a, t) \)) multiplied by powers of \( t \), can be expressed as linear combination of Laguerre functions, i.e.,

\[
\int t^p L_q(a, t) = \sum_{i=0}^{q} c_i L_i(a, t),
\]

where coefficients \( c_i = \{c_i, \ i = 0, 1, \ldots, q\} \) are explicitly computed in matrix form as

\[
\tilde{c} = H b
\]

with

\[
H(i, j) = (-1)^{i+j} \binom{a + j}{j - i}, \quad i, j = 0, 1, \ldots, q
\]

and

\[
b(i) = \binom{q + a}{q - i} \binom{k + i}{k + i + p}!, \quad i = 0, 1, \ldots, q.
\]

This yields a relation between Laguerre basis functions and elementary symmetric functions on system poles, whose properties could be investigated. Moreover the approach proposed in this paper allows to work directly in the Laplace domain by using formula (6) which involves derivatives of \( Y(s) \) and elementary symmetric functions on system poles.

Although reduced models are found to be stable, in general, our approach does not guarantee a stable reduced model. This could represent a problem if a stable model is required. We firmly believe that the stability of the reduced model is connected, in some way, to the value of \( m, n \) and \( T_s \) and it could be interesting to investigate this relation although such a theoretical investigation is beyond the scope of this paper. If a stable system is required then any optimization procedure can be used to solve (12) subject to constraints on \( \sigma(n_p, i) \) in order to guarantee that roots of polynomial

\[
Q(z) = z^{n_p} + \sum_{i=1}^{n_p} \sigma(n_p, i) z^{n_p - i}
\]

have negative real part. Such constraints can be easily found, for example, by the well-known Routh’s criterion. Nevertheless we emphasize that the purpose of this paper is to obtain a reduced model without iterative optimization. Furthermore our approach can be useful to find, in a fast way, a starting point for a next optimization step, reducing the searching space.

### References


