Model Matching of SISO Neutral Time Delay Systems via Output Feedback

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Abstract—The necessary and sufficient conditions for the solvability of the exact model matching problem for general neutral single input – single output multi-delay systems, via a realizable dynamic output feedback and a realizable dynamic precompensator, are established. The general class of the realizable dynamic controllers solving the problem is analytically determined. The results are successfully applied to the model of a cascade connection of two mixers as well as a numerical example.

I. INTRODUCTION

The problem of exact model matching has attracted considerable attention for the special case of retarded single delay systems in [1]-[4]. All results in the field focused towards a realizable controller, namely a controller not involving predictions. In [5] the problem has been solved for neutral single delay systems, via realizable but not proportional (dynamic) controllers in the form of dynamic precompensators. In [7] the problem of exact model matching for the general class of left invertible multi delay neutral time delay systems is extensively solved using a realizable proportional (not dynamic) state and output feedback.

Motivated by the results in [6] we studied the problem of exact model matching for the general class of left invertible multi delay neutral time delay systems using a realizable dynamic feedback and a realizable dynamic precompensator. The necessary and sufficient conditions for the solvability of the aforementioned problem are established. The general solution of the respective realizable controller elements is also derived. The results are successfully applied to the model of a cascade connection of two mixers as well as a numerical example.

The present results cover the solution of the exact model matching problem for the general class of left-invertible retarded multi-delay systems while other special types of neutral or retarded delay systems (i.e. commensurate delays) are also covered by the present results.

II. SISO LINEAR MULTI-DELAY SYSTEMS

Consider the general class of single input – single output (SISO) linear neutral multi-delay differential systems

\[ \sum_{j=1}^{\infty} \hat{E}_j \{ t - \sum_{i=1}^{q_j} \tau_i \} = \sum_{j=1}^{\infty} \hat{A}_j \{ t - \sum_{i=1}^{q_j} \tau_i \} + \sum_{j=1}^{\infty} \hat{b}_j \{ t - \sum_{i=1}^{q_j} \tau_i \} \] (1a)

\[ \sum_{j=1}^{\infty} \hat{c}_j \{ t - \sum_{i=1}^{q_j} \tau_i \} = \sum_{j=1}^{\infty} \hat{c}_j \{ t - \sum_{i=1}^{q_j} \tau_i \} \] (1b)

where \( x(t) \in \mathbb{R}^n \) denotes the vector of state variables, \( u(t) \in \mathbb{R} \) the vector of control inputs, \( y(t) \in \mathbb{R} \) the performance output, \( \tau_i \) \( (i = 1, \ldots, q) \) are positive real numbers denoting point delays, and \( q_{ij} \) \( (j = 1, \ldots, q_0 ; i = 1, \ldots, q) \) is a finite sequence of integers with regard to \( i \) and \( j \). The quantities \( q \) and \( q_0 \) are positive integers.

Clearly, if the quantity \( \sum_{i=1}^{\infty} q_{ij} \tau_i \) is negative then it denoted prediction. The real matrices \( \hat{E}_j, \hat{A}_j, \hat{b}_j \) have \( n \) rows while the real matrices \( \tau_i, \hat{c}_j \) have one row.

The interest is focused on the forced behavior of the system, i.e. for zero initial and past conditions \( x(t) = 0, u(t) = 0 \) for \( t < 0 \). Defining

\[ T = \begin{bmatrix} \tau_1 & \cdots & \tau_q \end{bmatrix} \]

\[ e^{st} = \begin{bmatrix} \exp(-st_1) & \cdots & \exp(-st_q) \end{bmatrix} \]

the system (1) can be described in the frequency domain by the following set of equations.
\[ s\hat{E}(e^{sT})X(s) = \hat{A}(e^{sT})X(s) + \hat{b}(e^{sT})U(s) \quad (2a) \]
\[ \tau(e^{sT})Y(s) = \hat{c}(e^{sT})X(s) \quad (2b) \]

where \( X(s) = L\{x(t)\}, \quad U(s) = L\{u(t)\}, \quad Y(s) = L\{y(t)\} \) with \( L\{\cdot\} \) be the Laplace transform of the argument signal, while

\[
\hat{E}(e^{sT}) = \sum_{i=1}^{N} \hat{E}_i \exp\{-s\left[\sum_{j=1}^{q} q_{ij}\tau_i\right]\}
\]
\[
\hat{A}(e^{sT}) = \sum_{i=1}^{N} \hat{A}_i \exp\{-s\left[\sum_{j=1}^{q} q_{ij}\tau_i\right]\}
\]
\[
\hat{b}(e^{sT}) = \sum_{i=1}^{N} \hat{b}_i \exp\{-s\left[\sum_{j=1}^{q} q_{ij}\tau_i\right]\}
\]
\[
\tau(e^{sT}) = \sum_{i=1}^{N} \tau_i \exp\{-s\left[\sum_{j=1}^{q} q_{ij}\tau_i\right]\}
\]
\[
c(e^{sT}) = \sum_{i=1}^{N} \hat{c}_i \exp\{-s\left[\sum_{j=1}^{q} q_{ij}\tau_i\right]\}
\]

where \( \exp[\cdot] = e^{\cdot} \) is the exponential of the argument quantity. The system of equations in (2) can be expressed in normal system form as follows

\[ sX(s) = A(e^{sT})X(s) + b(e^{sT})U(s) \quad (3a) \]
\[ Y(s) = c(e^{sT})X(s) \quad (3b) \]

where \( A(e^{sT}) = \left[\hat{E}(e^{sT})\right]^{-1}\hat{A}(e^{sT}), \quad b(e^{sT}) = \left[\hat{E}(e^{sT})\right]^{-1}\hat{b}(e^{sT}), \quad c(e^{sT}) = \frac{\hat{c}(e^{sT})}{\tau(e^{sT})} \) while the open loop transfer matrix takes on the form

\[ h(s, e^{sT}) = c(e^{sT})\left[sI_n - A(e^{sT})\right]^{-1}b(e^{sT}) \quad (4) \]

where \( \Omega(s) \) is the external input. The feedback and precompensator gains \( k(s, e^{sT}) \) and \( g(s, e^{sT}) \) are rational functions of \( s \). The respective numerator and denominator polynomial coefficients are multivariable rational functions of \( e^{-\tau_1}, \ldots, e^{-\tau_q} \).

Substituting the control law (5) to the system (3) it can be observed that the closed loop transfer matrix takes on the form

\[ h_c(s, e^{sT}) = c(s, e^{sT}) \times \left[sI_n - A(e^{sT}) - B(e^{sT})k(s, e^{sT})c(e^{sT})\right]^{-1} \times b(e^{sT})g(s, e^{sT}) \quad (6) \]

while the problem of exact model matching is formally stated as in the following equation

\[ h_c(s, e^{sT}) = m(s, e^{sT}) \quad (7a) \]

or equivalently

\[ c(e^{sT})\left[sI_n - A(e^{sT}) - B(e^{sT})k(s, e^{sT})c(e^{sT})\right]^{-1} \times b(e^{sT})g(s, e^{sT}) = m(s, e^{sT}) \quad (7b) \]

where \( m(s, e^{sT}) \) is the closed-loop transfer function which is a rational function of \( s \) with numerator and denominator polynomials having coefficients that they are multivariable rational functions of \( e^{-\tau_1}, \ldots, e^{-\tau_q} \) (or more compactly of \( e^{sT} \)). For the problem at hand to be solvable via implementable controller it is necessary for the controller elements \( k(s, e^{sT}) \) and \( g(s, e^{sT}) \) to be realizable [5] and [6]).

IV. SOLUTION OF THE PROBLEM

In what follows, we distinguish two cases for the closed loop transfer function \( m(s, e^{sT}) \). The first case is \( m(s, e^{sT}) = 0 \) for every \( s \) (i.e. \( m(s, e^{sT}) \equiv 0 \)) while the second is \( m(s, e^{sT}) \neq 0 \). It is obvious that in the first case the problem is always solvable with \( k(s, e^{sT}) \) arbitrary and \( g(s, e^{sT}) \equiv 0 \). In the second case, relation (7b) can be rewritten as

\[ c(e^{sT})\left[sI_n - A(e^{sT})\right]^{-1}b(e^{sT}) = m(s, e^{sT}) \times [g(s, e^{sT})]^{-1} \times \left\{1 - k(s, e^{sT})c(e^{sT})\right\} \]

or equivalently
\[ g(s, e^{sT}) = \left[ \left( h(s, e^{sT}) \right)^{-1} - k(s, e^{sT}) \right] m(s, e^{sT}) \]  

(8)

It is obvious that the open loop transfer function and the precompensator must be invertible.

The model’s and the open loop transfer functions can be rewritten as  

\[ m(s, e^{sT}) = e^{-m^*} m'(s, e^{sT}) \] 

and  

\[ h(s, e^{sT}) = \left( e^{-m^*} \right) h'(s, e^{sT}) \] 

where \( m'(s, e^{sT}) \) and \( h'(s, e^{sT}) \) are birealizable rational functions and \( r_m \) and \( r_h \) are appropriate reals. Using the above definitions relation (8) can be rewritten as

\[ \hat{g}(s, e^{sT}) = e^{-m(s-\gamma)} - e^{-m^*} \hat{k}(s, e^{sT}) \]  

(9)

Where

\[ \hat{k}(s, e^{sT}) = k(s, e^{sT})h'(s, e^{sT}) \]  

(10a)

\[ \hat{g}(s, e^{sT}) = g(s, e^{sT})\left[ m'(s, e^{sT}) \right]^{-1} h'(s, e^{sT}) \]  

(10b)

From (10) it can be observed that \( g(s, e^{sT}) \) and \( k(s, e^{sT}) \) are realizable if and only if \( \hat{g}(s, e^{sT}) \) and \( \hat{k}(s, e^{sT}) \) are realizable. The necessary and sufficient conditions for the problem to be solvable via realizable controller will be presented in Theorem 3.1 while the general form of the realizable controller will be presented in Corollary 3.1.

**Theorem 3.1**: The problem of exact model matching for the general class of left invertible multi delay neutral time delay systems via realizable controller with dynamic output feedback and dynamic precompensator is solvable if and only if

\[ r_h \leq r^* \]  

(11)

where

\[ r^* = \begin{cases} r_m, & \text{if } r_m \geq 0 \\ 0, & \text{if } r_m < 0 \end{cases} \]

**Proof**: We distinguish two cases. The first is \( r_m \geq 0 \) while the second is \( r_m < 0 \). In the first case since \( \hat{k}(s, e^{sT}) \) must be realizable it is observed that

\[ e^{-m^*} \hat{k}(s, e^{sT}) \] 

is also realizable. Hence, according to (9) for \( \hat{g}(s, e^{sT}) \) to be realizable it is necessary for \( e^{-m(s-\gamma)} \) to be also realizable or equivalently that

\[ r_m \geq r_h \]  

(12)

Inversely if (12) is satisfied then using (9) and choosing \( \hat{k}(s, e^{sT}) \) to be realizable a realizable \( \hat{g}(s, e^{sT}) \) is derived. In the second case, i.e. \( r_m < 0 \), we distinguish two subcases. The first is \( r_m \geq r_h \) while the second is \( r_m < r_h \). In the first subcase, where it must also hold that \( r_h < 0 \), we observe that \( e^{-m(s-\gamma)} \) is realizable. Hence, for

\[ \hat{g}(s, e^{sT}) \] 

and \( \hat{k}(s, e^{sT}) \) to be realizable it is necessary and sufficient to have a realizable \( e^{-m^*} \hat{k}(s, e^{sT}) \) via a realizable \( \hat{k}(s, e^{sT}) \). This can always be satisfied if we choose \( \hat{k}(s, e^{sT}) \) to be enough realizable i.e. choose \( e^{-m^*} \hat{k}(s, e^{sT}) \) to be realizable. In the second subcase \( r_m < r_h \) it is observed that for \( \hat{g}(s, e^{sT}) \) to be realizable it is necessary that (see (9)) \( r_m \leq r_m - r_h \) or equivalently that

\[ r_h \leq 0 \]  

(13)

Assuming that \( r_h \leq 0 \) and choosing \( \hat{k}(s, e^{sT}) = e^{-m^*} \hat{k}(s, e^{sT}) \) where \( \hat{k}(s, e^{sT}) \) is realizable then \( \hat{k}(s, e^{sT}) \) is also realizable and

\[ \hat{g}(s, e^{sT}) = g(s, e^{sT}) \] 

(14a)

\[ \hat{g}(s, e^{sT}) = g(s, e^{sT})\left[ m'(s, e^{sT}) \right]^{-1} h'(s, e^{sT}) \]  

(14b)

where

\[ g(s, e^{sT}) = \begin{cases} e^{-m(s-\gamma)} - e^{-m^*} \hat{k}(s, e^{sT}) & \text{if } r_m \geq 0 \& r_m \geq r_h \\ -e^{-m(s-\gamma)} - \hat{k}(s, e^{sT}) & \text{if } r_m < 0 \& r_m \geq r_h \end{cases} \]

\[ \hat{k}(s, e^{sT}) = \begin{cases} \text{arbitrary realizable} & \text{if } r_m \geq 0 \& r_m \geq r_h \\ e^{-m^*} \hat{k}(s, e^{sT}) & \text{if } r_m < 0 \& r_m \geq r_h \\ e^{-m^*} - \hat{k}(s, e^{sT}) & \text{if } r_m < 0 \& r_m < r_h \end{cases} \]

Corollary 3.1: Assume that the condition of Theorem 3.1 is satisfied and the model’s transfer function is invertible. Then the general form of the controller elements \( k(s, e^{sT}) \) and \( g(s, e^{sT}) \), solving the problem of exact model matching of SISO neutral multi delay differential systems is given by the relations

\[ k(s, e^{sT}) = \hat{k}(s, e^{sT}) \] 

(14a)

\[ g(s, e^{sT}) = \hat{g}(s, e^{sT}) \]  

(14b)
V. NUMERICAL EXAMPLE

To demonstrate the above proposed technique, consider the set of multi-delayed system equations with \( n = 2 \)

\[
\begin{align*}
\dot{x}_1(t) + \dot{x}_2(t) + \dot{x}_3(t) &= x_1(t) + x_3(t) + u(t) \\
\dot{x}_2(t) + \dot{x}_3(t) &= x_2(t) + u(t)
\end{align*}
\]

(15a)

\[
\begin{align*}
\frac{1}{2} y(t) + g(t) &= x_1(t) + x_2(t) + x_3(t)
\end{align*}
\]

(15c)

This is of the form (1) with \( q_0 = 5, \ q = 2, \ q_1 = 2, \ q_2 = 1, \ q_3 = 3, \ q_4 = 2, \)

\[
\begin{bmatrix}
\hat{E}_1 \\
\hat{E}_2 \\
\hat{E}_3 \\
\hat{E}_4
\end{bmatrix} =
\begin{bmatrix}
1 & 1 \\
0 & 1 \\
0 & 1 \\
0 & 1
\end{bmatrix},
\begin{bmatrix}
\hat{A}_1 \\
\hat{A}_2 \\
\hat{A}_3 \\
\hat{A}_4
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
\begin{bmatrix}
\hat{B}_1 \\
\hat{B}_2 \\
\hat{B}_3 \\
\hat{B}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix},
\begin{bmatrix}
\hat{C}_1 \\
\hat{C}_2 \\
\hat{C}_3 \\
\hat{C}_4
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

After applying some manipulations the open loop transfer function is computed to be

\[
h(s, e^{-st}) = \frac{b_1(e^{-st}) s + b_0(e^{-st})}{s^2 + a_1(e^{-st}) s + a_0(e^{-st})}
\]

where

\[
\begin{align*}
b_1(e^{-st}) &= \frac{2 e^{\frac{3}{2} s t + \frac{1}{2} s^2 t}}{\left[2 + 3 e^{\frac{3}{2} s t} + e^{\frac{3}{2} s^2 t}\right] s + a_1(e^{-st})}, \\
b_0(e^{-st}) &= \frac{2 e^{\frac{3}{2} s t + \frac{1}{2} s^2 t}}{\left[2 + 3 e^{\frac{3}{2} s t} + e^{\frac{3}{2} s^2 t}\right] s + a_0(e^{-st})},
\end{align*}
\]

It can readily be observed that \( h(s, e^{-st}) \) is birealizable, i.e. \( h(s, e^{-st}) = h^*(s, e^{-st}) \) or equivalently \( r_\gamma = 0 \). Assume that \( m(s, e^{-st}) = \frac{1}{s + e^{-st}} \). It can be observed that \( m(s, e^{-st}) \) is also birealizable, i.e. \( m(s, e^{-st}) = m^*(s, e^{-st}) \) or equivalently \( r_\gamma = 0 \). Using Theorem 3.2, the general form of the controller rational functions take on the form

\[
\tilde{g}(s, e^{-st}) = 1 - \hat{k}(s, e^{-st})
\]

where \( \hat{k}(s, e^{-st}) \) is arbitrary realizable. Clearly \( \hat{k}(s, e^{-st}) \) must be different than 1 so that \( \tilde{g}(s, e^{-st}) \) is different than zero. Let

\[
\hat{k}(s, e^{-st}) = \frac{1}{s + 1}
\]

Equivalently, using (14a) we get

\[
k(s, e^{-st}) = \frac{s^2 + a_1(e^{-st}) s + a_0(e^{-st})}{(s + 1)[b_1(e^{-st}) s + b_0(e^{-st})]}
\]

(17)

Furthermore, using (14b) we get

\[
g(s, e^{-st}) = \tilde{g}(s, e^{-st}) m^*(s, e^{-st}) [h^*(s, e^{-st})]^{-1}
\]

or equivalently

\[
g(s, e^{-st}) = \frac{s [s + a_1(e^{-st}) s + a_0(e^{-st})]}{(s + 1)(s + e^{-st})[b_1(e^{-st}) s + b_0(e^{-st})]}
\]

(18)

Combining (6), (17) and (18) we get

\[
b_1(s, e^{-st}) = \frac{1}{s + e^{-st}}
\]
VI. APPLICATION TO A CASCADE CONNECTION OF TWO MIXERS

Consider the cascade connection of two fully filed mixers (see [7]) according to the scheme presented in Figure 1, where \( e_m(t) \) is the input concentration of the product, \( Q \) is the constant flow intensity for \( e_m(t) \), \( V \) is the volume of Mixers 1 and 2, \( L \) denotes the length of the reactor and \( \tau \) is a constant delay arising in the reactor.

![Figure 1. Scheme of a Cascade Connection of Two Mixers](image)

Defining

\[
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix} =
\begin{bmatrix}
    c_1(t) \\
    c_2(t)
\end{bmatrix},
\quad u(t) = e_m(t)
\]

the mathematical model of the dynamical system with delay in the state takes on the form

\[
\begin{align*}
\dot{x}(t) &= A_1 x(t) + A_2 x(t-\tau) + B_1 u(t) \\
g(t) &= c x(t)
\end{align*}
\]

where

\[
A_1 = \begin{bmatrix}
-\frac{Q}{V} & 0 \\
0 & -\frac{Q}{V}
\end{bmatrix},
A_2 = \begin{bmatrix}
0 & 0 \\
-\frac{Q}{V} & 0
\end{bmatrix},
B_1 = \frac{Q}{V},
\]

\[
c = \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

Applying elementary computations, the transfer function of the system takes on the form

\[
h(s, e^{-\tau}) = \frac{e^{-\tau} \left( \frac{Q}{V} \right)^2}{\left( s + \frac{Q}{V} \right)^2}
\]

The open loop transfer function can be rewritten as

\[
h(s, e^{-\tau}) = e^{-\tau} h^*(s, e^{-\tau})
\]

where \( r_n = \tau \) and

\[
h^*(s, e^{-\tau}) = \frac{Q}{V} \left( \frac{Q}{s + \frac{Q}{V}} \right)^2
\]

Clearly \( h^*(s, e^{-\tau}) \) is birealizable. Let

\[
m(s, e^{-\tau}) = \frac{e^{-\tau} \lambda_1 \lambda_2 \left( \frac{Q}{V} \right)^2}{\left( s + \frac{Q}{V} \right) \left( s + \frac{Q}{V} \lambda_1 \right)}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are positive reals. Equivalently, the closed loop transfer function can be rewritten as

\[
m^*(s, e^{-\tau}) = e^{-\tau} m^*(s, e^{-\tau})
\]

where \( r_n = \tau \) and

\[
m^*(s, e^{-\tau}) = \frac{\lambda_1 \lambda_2 \left( \frac{Q}{V} \right)^2}{\left( s + \frac{Q}{V} \lambda_1 \right) \left( s + \frac{Q}{V} \lambda_2 \right)}
\]

where \( m^*(s, e^{-\tau}) \) is also birealizable. Clearly, the controller transfer functions take on the form

\[
\begin{align*}
k(s, e^{-\tau}) &= \hat{k}(s, e^{-\tau}) [h^*(s, e^{-\tau})]^{-1} \\
g(s, e^{-\tau}) &= \hat{g}(s, e^{-\tau}) m^*(s, e^{-\tau}) [h^*(s, e^{-\tau})]^{-1}
\end{align*}
\]

Choosing

\[
\begin{align*}
\hat{k}(s, e^{-\tau}) &= \frac{1}{(s + \rho_1)(s + \rho_2)} \\
\hat{g}(s, e^{-\tau}) &= 1 - \frac{e^{-\tau}}{(s + \rho_1)(s + \rho_2)}
\end{align*}
\]

where \( \rho_1 \) and \( \rho_2 \) are positive reals, and combining (22), (25), (26) and (27), the controller transfer functions take on the final form

\[
k(s, e^{-\tau}) = \frac{\left( \frac{Q}{V} \right)^2 \left( s + \frac{Q}{V} \right)^2}{(s + \rho_1)(s + \rho_2)}
\]
\[ g(s,e^{-\tau}) = \frac{\lambda_1 \lambda_2 \left( s + \frac{Q}{V} \right)^2}{\left( s + \frac{\rho_1 Q}{V} \right) \left( s + \frac{\rho_2 Q}{V} \right) \left( s + \rho_1 \right) \left( s + \rho_2 \right)} \times \left[ s^2 + (\rho_1 + \rho_2) s + (\rho_1 \rho_2 - e^{-\tau}) \right] \]  

(28b)

yielding to the desired closed loop transfer function

\[ h(s,e^{-\tau}) = \frac{e^{-\tau} \lambda_1 \lambda_2 \left( \frac{Q}{V} \right)^2}{\left( s + \frac{\rho_1 Q}{V} \right) \left( s + \frac{\rho_2 Q}{V} \right)} \]

From (23) and (25) it can readily be observed that by appropriate choice of \( \lambda_1 \) and \( \lambda_2 \) the rise time of the closed loop system can be reduced significantly. Furthermore, the closed loop steady state gain remains unity. For example, letting \( \lambda_1 = 2 \) and \( \lambda_2 = 4 \), the rise time (95% of the steady state value) of the closed loop system can be computed to be

\[ t_{95\%} = \frac{1.838V}{Q} + \tau \]  

(29a)

while for the rise time of the open loop system is computed to be

\[ t_{95\%} = \frac{4.743V}{Q} + \tau \]  

(29b)

Clearly it holds that \( \frac{t_{95\%} - \tau}{t_{95\%} - \tau} = 2.58 \). This way it can be considered that the closed loop system is 2.58 times faster than the open loop.

VII. CONCLUSIONS

The necessary and sufficient conditions for the solvability of the exact model matching problem for general neutral single input – single output multi-delay systems via realizable dynamic output feedback and realizable dynamic precompensator controllers have been established. The general solution of the realizable dynamic controllers solving the problem have been derived. The results have successfully been applied to the model of a cascade connection of two mixers as well as a numerical example. The results cover the solution of the exact model matching problem for the general class of left-invertible retarded multi-delay systems while other special types of neutral or retarded delay systems are also covered by the present results.

The form of the derived conditions and the general expression of the controllers facilitate the extension of the present results to an adaptive scheme.

REFERENCES


