Exact fault and disturbance decoupling by means of direct input reconstruction and estimation of the inverse dynamics

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Abstract—The idea of inversion-based direct input reconstruction for robust detection and separation of multiple, possibly simultaneous faults in the presence of external, non-mutually separable disturbances for linear dynamical systems was presented in [1]. In this short paper this concept is pursued further: it is shown how in a specific filtering structure, relying on the inverse representation of the system, a Luenberger state observer can be designed providing estimation of the states of the inverse-based residual generator reverting to disturbance decoupled detection residuals with exact fault separation. This result can be considered as a corollary of the approximate disturbance attenuating solution presented in [1].

I. INTRODUCTION
The distinctive term direct input reconstruction is used to identify a relatively new idea that has been proposed and investigated in the methodology of the design of detection filters for detection, separation and estimation of faults in linear as well as in nonlinear systems quite lately. This approach is an application of dynamic inversion to filtering which is dual to the concept of dynamic inversion for control. The difference between these inversion approaches is that control uses a right inverse whereas estimation uses a left inverse of the system.

The method arrives at detector architectures whose residual outputs consist of the fault signals while their inputs are the standard observables (inputs and outputs) of the system and possible their time derivatives. The approach makes not only the detection and isolation but also the estimation of the fault signals possible as it was rendered in [2] in an algebraic and in [3], [4] in a geometric approach, respectively.

The effectiveness and distinctive capabilities of the direct input reconstruction method applied to robust fault detection and isolation was recently demonstrated in [1]. It was shown, how advanced methods of detection filter design, such as inversion-based residual generation and \(H_\infty\) optimal filtering, and the novel combination of them, may contribute to the solution of earlier not solvable problems. Namely, direct fault reconstruction combined with an \(H_\infty\) filter was applied to disturbance attenuated fault decoupling.

The idea of inversion-based direct input reconstruction for robust detection, estimation and separation of multiple, possibly simultaneous faults was presented and demonstrated in [1] by considering the linear dynamical system subject to faults and external disturbances:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Bd_1 + L_2f_2 \\
y_1 &= C_1x + Du + Dd_1 + Mf_1 \\
y_2 &= C_2x + D_2u + Dd_2, 
\end{align*}
\]

where \(M\) is full rank with \(x \in \mathbb{R}^n, u \in \mathbb{R}^m\) and \(y_1 \in \mathbb{R}^p_1\) and \(y_2 \in \mathbb{R}^p_2\), where the rest of the matrices of the system representation are given in the appropriate dimensions. The unknown, bounded time functions \(f_1(t), f_2(t), d(t)\) represent the actuator and sensor faults, moreover, the disturbance, respectively. An underlying working assumption in [1] was that faults and disturbances in (1)–(3) were non-mutually separable in terms of the separability conditions of the traditional filter design techniques such as geometric decoupling or unknown input observer design methods.

The objective was to construct a robust residual generator in the state space, that shows up the effects of \(f(t)\) at its output irrespectively of the presence of the disturbance input \(d(t)\) so that the separation between the respective fault transmission levels is maintained making the detection and isolation robustly possible. Because of the presence of the non-separable disturbance in the system, the variety of applicable solution approaches, available in the literature, is traditionally confined to methods providing suppression of the disturbance effects on the filter’s residual.

Two approaches of this type were presented and contrasted to each other in [1]. One of these solutions was obtained with using traditional \(H_\infty\) filtering providing optimal disturbance suppression. As it was shown, this solution suffers from a tendency of poor fault separation performance in general. For the enhancement of fault selectivity of the filter, a solution approach, based on the idea of inversion-based direct input reconstruction combined with traditional \(H_\infty\) filtering was proposed as a solution alternative.

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In this paper this novel idea is pursued further and is shown how this method may provide robust detection and fault signal estimation with exact fault and disturbance separation in contrast to the approximate decoupling result given in [1].

Consider system (1)-(3) subject to faults and disturbances and suppose that the system from the faults to the given outputs is left invertible, i.e., one has

\[ f_1 = C_f x + B_{f1} y_1 + B_{af} u \]  
\[ f_2 = C_f x + B_{f2} \xi_2 + B_{af} \xi_u \]  

provided that \( d = 0 \) and \( f = [f_1, f_2]^T \) and

\[ \xi_f = [y, \dot{y}, \ldots, y^{(k-1)}]^T, \quad \xi_u = [u, \dot{u}, \ldots, u^{(k)}]^T \]

where \( k \) is the maximum relative degree of the system (1) and (3), corresponding to the pair \((f_2, y_2)\), for details see [4]. In a classical approach the unknown state in (4)-(5) is expressed in terms of the derivatives of the output functions and the components of the possible zero dynamics. Since the initial conditions are not known, this approach might provide a reliable solution only in a closed-loop manner (see e.g., [5]). As another condition, the zero dynamics of the system is required to be stable. Substituting (4)-(5) to the state equation (1) one obtains the inverse dynamics

\[ \dot{x} = \hat{A} x + B_{f} \xi_f + B_{af} \xi_u + B_d d \]  

that provides a useful structure for constructing a filter for generating detection residuals as shown in Fig. 1.

Assuming that additional measurements are available, e.g.,

\[ y_3 = C_z x + D_{u,3} u + D_{d,3} d, \]

such that the system with output functions \( \bar{y} = [y_2, y_3]^T \) is observable it is possible to construct a state observer for the estimation of the state \( x \) of (6). Relying on this estimation one can obtain the reconstructed fault signals by using (4)-(5).

![Fig. 1. Direct input (fault) reconstruction by means of system inversion; \( \Sigma \) is the plant, \( \mathcal{D} \) is the reconstructor or detector which can be obtained as the (left) inverse \( \Sigma^{-1} \) of the system.]

**Proposition 1:** Assume the system (1) is left invertible w.r.t. each of the unknown inputs \( f(t) \), \( d(t) \) to be detected and separated from each other and construct the inverse system resulting in the respective forms (4-6) (cf. with the solution presented in [1] where the inversion was made w.r.t. the fault signals only). Define the new output functions \( \bar{y} = \hat{C} x, \quad \bar{y} = \{ y_j \}_{j \in \{1, \ldots, p\}, j \neq i}, \quad \text{dim} \bar{y} < \text{dim} y \), which, due to \( \text{dim} y > \text{dim} f \), are not parts of the calculation of the inverse in (4)-(5), i.e., the new observations \( C \) are composed of selected rows of \( C \). Consider the representation (4-6) equipped with the new measurement functions \( \bar{y} \) and assume the pair \((\hat{A}, \bar{C})\) observable. Then, a state observer can be designed to get an estimate of the unknown state in (6). Alternatively, if the pair \((\hat{A}, \bar{C})\) is found non-observable \( \bar{y}(t) \) can be extended with one or more of the original measurements \( \{ y_j \} \) attempting to construct an observable representation.

The solution method relying on the above proposition is presented in the next section on basis of the application example patterned after [1]. This example considers the design of an F16XL aircraft fault detection filter that monitors an elevon actuator and a normal accelerometer sensor in the presence of persistent wind gust disturbance where the disturbance could not be decoupled from the faults using traditional \((C,A)\)-invariant subspace design, and could not be attenuated either while keeping fault effects decoupled as it was originally discussed in [6], [7].

**II. THE F16XL AIRCRAFT MONITORING PROBLEM**

A reduced-order aircraft model, (see [1]), linearized about trimmed level flight at 10.000 ft (3048 m) and speed M0.9, representing the longitudinal dynamics only including a first-order wind gust model with faults can be considered as

\[ \dot{x} = Ax + B_{f} \theta + B_{f} (\delta + \nu), \]

\[ y = C x + E_{A} \mu_{A}, \]  

where the elevon deflection angle \( \delta(t) \) is taken as input, and the normal accelerometer sensor fault \( \mu_{A}(t) \), the elevon fault as an actuator fault \( v_{\delta}(t) \) and the wind gust disturbance \( \omega(t) \) are arbitrary time-varying real scalars. Note that the elevon fault enters the system in the same direction as the input does and the port and starboard elevons are modeled as a slaved system: no lateral dynamics and no elevon actuator dynamics are taken into consideration. The state and input/output variables of the system contained in system model (7) are summarized in Table 1 and 2. The parameters of the system are given by the matrices

\[
A = \begin{bmatrix}
-0.6074 & 0.0430 & -0.8886 & -0.5587 & 0.0430 \\
0.0205 & -1.4666 & 16.5800 & -0.0299 & -1.4666 \\
0.1377 & -1.6788 & -0.6189 & 0 & -1.6788 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.1948 & 0
\end{bmatrix},
\]

\[
B_{\theta} = \begin{bmatrix}
0 \\
0 \\
0.157 \\
0 \\
1.57
\end{bmatrix}, \quad B_{\delta} = \begin{bmatrix}
-0.1672 \\
-1.5179 \\
-9.7842 \\
0 \\
0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
0 & 0 & 1.0000 & 0 & 0 \\
0 & 0.0591 & 0 & 0 & 0.0591 \\
0.0139 & 1.0517 & 0.1485 & -0.0299 & 0 \\
-0.0677 & 0.0431 & 0.0171 & 0 & 0
\end{bmatrix}.
\]
and $E_k = [0\ 0\ 1\ 0\ 0]^T$. For further notational convenience, let $A = [A'_1 A'_2 \cdots A'_3]^T$ and $C = [c'_1 c'_2 c'_3]^T$, where $A_i$ and $c_i$ correspond to the $i$-th rows of $A$ and $C$, respectively. Following the technique and considerations presented in [1], the system (7) can be converted to the representation

$$
\dot{x} = Ax + F_{A_k} \mu_{A_k} + B_8 (\delta + \nu_8) + B_{90} \omega, \quad y = Cx, \quad (8)
$$

with $[F_{A_k}\ F_{A_k}]$, where $m_{A_k}$ is a fictitious signal representing the sensor fault effect and the directions $F_{A_k}$ and $F_{A_k}$ are given by $E_k = CF_{A_k}$ and $F_{A_k} = AF_{A_k}$ so that

$$
F_{A_k} = \begin{bmatrix}
0.01 & 0 & 0 & 0 \\
0.0143 & 0 & 0 & 0 \\
0.0 & 0.245 & 0 & 0 \\
0 & 0 & 0.245 & 0
\end{bmatrix}^T.
$$

Due to Proposition 1 the realization of the full decoupling idea requires the calculation of the inverse w.r.t. each of the unknown inputs $\mu_{A_k}(t)$, $\nu_8(t)$ and $\omega(t)$.

(i) Inversion for $\mu_{A_k}$: The only measurement available for the determination of the accelerometer fault is the third output equation $y_3 = c'_2 x + \mu_{A_k}$, from which the sensor fault can be expressed by inversion, i.e.,

$$
\mu_{A_k} = y_3 - c'_2 x. \quad (9)
$$

(ii) Inversion for $\nu_8$: Because it would be not a wise idea to include the derivative of the disturbance function in the residual, let the inverse calculation w.r.t. $\nu_8$ be based on the derivative of the fourth equation (8) is not included), letting $y_4 = c'_4 x = c'_4 Ax + c'_2 B_90 \omega + c'_3 B_8 (\delta + \nu_8)$. Since the equality $c'_4 B_{90} = 0$ holds, the actuator fault is obtained as

$$
\nu_8 = -\frac{c'_4 A}{c'_2 B_9} x + \frac{1}{c'_4 B_8} \dot{y}_4 - \delta. \quad (10)
$$

(iii) Inversion for $\omega$: For the reconstruction of the disturbance function one may choose between two opportunities: it can be based either on $y_1(t)$ or $y_2(t)$ (and also on their derivatives). Selecting the second measurement equation $y_2(t)$ it can be written that

$$
\begin{align*}
\dot{y}_2 &= c'_2 x \\
\ddot{y}_2 &= c'_2 \dot{x} = c'_2 (Ax + B_{90} \omega + B_8 (\delta + \nu)).
\end{align*} \quad (11)
$$

By substituting the known identity relation

$$
\delta + \nu_8 \triangleq (\dot{y}_4 - c'_2 Ax) \frac{1}{c'_4 B_8} \quad (12)
$$

$$
\begin{array}{|c|c|}
\hline
\text{TABLE I} & \text{STATE VARIABLES OF THE SYSTEM} \\
\hline
x_1 = \omega(t) & \text{longitudinal body axis velocity} \quad \text{ft/s} \\
x_2 = \nu(t) & \text{normal body axis velocity} \quad \text{ft/s} \\
x_3 = q(t) & \text{pitch rate} \quad \text{deg/s} \\
x_4 = \theta(t) & \text{pitch angle} \quad \text{deg} \\
x_5 = w(t) & \text{wind gust} \quad \text{ft/s} \\
\hline
\end{array}
$$

into $\ddot{y}_2$ one obtains the disturbance from (10) in the form

$$
\ddot{y}_2 = \frac{c'_4}{c'_4 B_{90}} \dot{y}_4 - \frac{c'_3}{c'_4 B_8} \frac{1}{c'_4 B_8} \dot{y}_4 - \delta. \quad (13)
$$

For the sake of a more compact system of notation let us introduce the identities

$$
\begin{align*}
\phi_1 &\triangleq \frac{1}{c'_4 B_{90}} \\
\phi_2 &\triangleq c'_4 \frac{1}{c'_4 B_8} c'_2 B_{90} \\
\phi_3 &\triangleq c'_3 \frac{1}{c'_4 B_8} \\
\phi \underline{4} &\triangleq \phi_3 + \phi_2. \quad (14)
\end{align*}
$$

By using (14-15) and substituting the new output functions (9), (10) and (13) into the state equation (7) one obtains the inverse dynamics $\dot{x} = \ddot{Ax} + B\ddot{u}$, by

$$
\ddot{A} = \left( I + B_90 (\phi_2 c'_4 - \phi_1 c'_2) + B_8 (\phi_3 \phi_1 c'_2 - \phi_2 c'_4) \right) A, \\
\ddot{B} = \begin{bmatrix} 0 & 0 & 0 & (\phi_1 B_{90} - \phi_3 B_8 B_90 - \phi_2 B_{90} + \phi_4 B_{90}) \end{bmatrix},
$$

$$
\ddot{u} = \begin{bmatrix} \delta \ y_3 \ y_4 \end{bmatrix}^T. \quad (15)
$$

Since in the present case the product $c'_2 B_{90}$ is obtained zero, the identities (14-15), as well as $\ddot{A}$ and $\ddot{B}$ reduce to

$$
\begin{align*}
\phi_1 &\triangleq \frac{1}{c'_4 B_{90}} \\
\phi_2 &\triangleq \frac{1}{c'_4 B_8} c'_2 B_{90} \\
\phi_3 &\triangleq 0, \\
\phi \underline{4} &\triangleq \frac{1}{c'_4 B_8}.
\end{align*}
$$

Relying on Proposition 1, consider the inverse system equipped with the measurement equation that was not utilized in the determination of the output functions $\mu_{A_k}(t), \nu_8(t), \omega(t)$ given by (9), (10) and (13), respectively, and write

$$
\dot{x} = \ddot{Ax} + B\ddot{u}, \quad \ddot{y} = \ddot{Cx} \quad (16)
$$

providing the residuals

$$
\begin{align*}
\begin{bmatrix}
\nu_8 \\
\omega \\
\mu_{A_k}
\end{bmatrix} &= \begin{bmatrix}
\begin{bmatrix} -c'_4 & c'_2 B_9 & -c'_3 \\
c'_4 & c'_2 B_9 & c'_3 B_{90} \\
c'_4 & c'_3 B_8 & c'_2 B_{90}
\end{bmatrix} A & + \\
\begin{bmatrix} -c'_4 \ B_{90} & 1 \\
c'_4 & c'_2 B_{90} & 1 \\
c'_4 & c'_3 B_8 & c'_2 B_{90}
\end{bmatrix} \begin{bmatrix} \delta \\
\dot{y}_3 \\
\dot{y}_4
\end{bmatrix}
\end{bmatrix}
\end{align*} \quad (17)
$$

$$
\begin{array}{|c|c|c|c|}
\hline
\text{TABLE II} & \text{INPUT/OUTPUT VARIABLES} \\
\hline
a = \delta(t) & \text{elevator deflection angle} \quad \text{deg} \\
y_1 = \phi(t) & \text{pitch rate} \quad \text{deg/s} \\
y_2 = \alpha(t) & \text{angle of attack} \quad \text{deg} \\
y_3 = A(t) & \text{normal acceleration} \quad \text{ft/s}^2 \\
y_4 = A(t) & \text{longitudinal acceleration} \quad \text{ft/s}^2 \\
\hline
\end{array}
$$

where, $\bar{y} \triangleq y_1$ and $\hat{C} \triangleq c'_1$ by definition. Based on the previous considerations, the following result is obtained by the application of Proposition 1.

If the system (16) is found observable, then a reduced order state observer can be designed to get an estimate $\hat{x}(t)$ of the state $x(t)$ of the inverse dynamics. Rendering $\hat{x}(t)$ available, it can be used for the calculation of the inverse outputs $v_8(t), \mu_4(t), \alpha(t)$ from (17). It can be easily checked, however, that the pair $(\hat{C}, \hat{A}) = (c'_1, A)$ in (16-17) is obtained non-observable. Due to Proposition 1, the original measurements (all but $y_3$) can be appended to $\bar{y}$, constructing the output matrix in the form $\hat{C} = [c'_1 \ c'_2 \ c'_3]'$. This newly constructed system is found already observable that makes the observer design problem viable.

The observer design problem i.e., the problem of finding a feedback matrix $K$, such that the closed loop observer gain $\hat{A} + K\hat{C}$ is stable and the observer satisfy some performance requirements (it has the desired set of eigenvalues) is a problem which has a number of standard solutions available which therefore is not included here. With the application of the eigenvalue assignment approach, a stable filter having acceptable transient behavior (by posing that filter eigenvalues along the real axis are smaller than $-0.5$), the observer gain $K$ and its respective spectrum $\sigma$ is obtained as

$$K = \begin{bmatrix}
20840 & -162670 & -70140 \\
32730 & -255510 & -110200 \\
30 & 20 & 10 \\
-2670 & 20790 & 8970 \\
-32730 & 255490 & 110210
\end{bmatrix},$$

$$\sigma(K) = \begin{bmatrix}
-1.3615 + 6.3015i \\
-1.3615 - 6.3015i \\
-1.0879 + 0.4244i \\
-1.0879 - 0.4244i \\
-0.7733
\end{bmatrix}.$$

The results of a continuous time process simulation assuming the simultaneous occurrence of an elevon fault and a normal accelerometer fault in the presence of a persistent wind gust disturbance are presented in Fig. 1-2. The result evidences that the filter gives the estimates of the fault signals $v_8$, and $\mu_4$, separately, disregarding the disturbance effect $\omega$ which, in this case, shows up in an independent residual direction, see Fig. 3. (Cf. with the disturbance attenuated solution of the same problem in [1]). For more practical considerations of this approach, see concluding remarks of [1].

III. Conclusions

In this paper the effectiveness of inversion-based direct input reconstruction for robust detection and separation of multiple simultaneous faults in the presence of external disturbances, which proved to be non-mutually separable with using traditional filter design methods, in linear dynamical systems was presented. It was shown how in a specific filtering structure a detection filter could be designed, providing exact fault and disturbance decoupling.

The filter was based on the left inverse of the system that can be formulated by using simple algebraic considerations. The basic contribution of this paper is to show that it is possible to use a state observer for the estimation of the unknown states of the inverse dynamics avoiding the complicated analytical solution process proposed in the literature for the calculation of the inverse such as e.g., in [3], [4]. In this paper a stable solution to this problem was obtained by using a Luenberger state observer.

Applicability of the method requires the availability of measurements in excess of the number of faults and the disturbance signals to be separated. Access to derivative measurements of some signals is also needed. The number and structure of available measurement data is an issue of system specification and design. In light of the development of recent sensor technology, direct
access to derivatives is a technical reality, evading the noise, generated by numerical differentiating methods. Though the issue of noise sensitivity coming from the noise amplification principle of time derived variables necessitates further investigation. Another very important direction of research is towards increasing robustness of the filter against system modeling errors which can make the inverse unstable.

REFERENCES


