Tropospheric signal delay estimation in repeat-pass SAR Interferometry with QR-factorization

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Abstract - In this paper we analyze the reason why the QR-factorization is a very useful method for estimating the atmospheric delay in the unwrapped step of the SAR interferometric processing. We compute the solution of phase delay parameter model and confront the result with the GPS data metrics and the result from the Saastamoinen model. It is important to develop the new method for estimating the atmospheric phase delay and errors unwrapping procedure without the GPS or metrological data.

I. INTRODUCTION

Atmospheric delay is variations in pressure, temperature and water vapor content cause changes in the effective reflective index along the ray path. Atmospheric delay in ratio signal propagation is known to be one of the major limitations of repeat-pass Interferometric Synthetic Aperture Radar (InSAR) [1, 2, 3, 4, 5].

After processing, the phase delay of a SAR image resolution element is the sum of several components, which are: 1) phase delay due to the location of the pixel on the ground, 2) phase delay due to the atmospheric contribution, 3) unwrapping procedure error.

In the suggested methodology, a pixel basis estimation of atmospheric delay will be performed. This paper investigates an approach for the estimation of atmospheric path delay based on the QR-factorization. This method was adopted because the maximum increment of the parameters used does not affect method’s accuracy. The method is very useful for solving the rank-deficient problems in the linear system of equations [6, 7, 8, 9, 10].

II. MATHEMATICAL BASIS

A QR-factorization of a real matrix A is A=QR where Q is orthogonal and R is upper triangular. Then, Ax=b can be solved using that Ax=b if \( Q^T Ax = Q^T b \), if \( Q^T QRx = Q^T b \) if \( DRx = Q^T b \) where \( D = Q^T Q \) is a diagonal matrix. Then DR is upper triangular matrix and x can be calculated by backward substitution. Different methods are proposed to compute the decomposition matrices Q and R. [6, 7, 8, 9, 10], in this paper we use the TLS method (Total Least Square).

III. THE PROPOSED METHOD

The Synthetic Aperture Radar (SAR) interferometric processing sequence includes a number of steps, one of which is the phase unwrapping whereby the phase ambiguities are solved. These unwrapping methods though, are dependent mainly on the effective length of the interferometric baseline and on atmospheric effects [11] producing inaccurate results for steep relief. For short baselines, although the unwrapped phase is more accurately estimated and the atmospheric effects are more directly interpretable on the interferograms, they reduce the topographic and deformation effect on the interferogram, leading to errors in height estimation. On the other hand, long baselines lead to a more accurate estimation of heights, but the atmospheric and deformation effect is hidden underneath [12].

Using the unwrapped phase obtained from multi-pass interferograms of approximately the same baseline length, the respective interferograms and coherence images as input units, the height values of the area, the atmospheric delay, as well as, errors due to unwrapping procedure will be modelled.

If in the interferograms that we examine, the baseline had the same length or minimum approximately, the phase delay due to topography in different interferograms is the same for the same topographic point.

i) Analysis

The method proposed for the assessment of atmospheric effects on InSAR is based on direct methods to application of QR-factorization. To include the all most of phase delay parameter in a functional model, which relates interferometric unwrapped phase to the unknown parameters of height, deformation, atmospheric delay, and the ambiguity of the phase [13,14].

\[
\Phi_i = \Phi_H + \Phi_\text{at} + \Phi_{\text{ERR}}
\]  

Where the \( \Phi_\text{at} \) is atmospheric effect, \( \Phi_H \) is connected with the parameters of height region and the phase delay
from the unwrapping procedure errors $\Phi_{ERR}$, and $\Phi_i$ is the phase delay result by the unwrapped process. Since it is very clear that in our disposition is the unwrapped phase delay and it linear relationship with three unknown described parameters for every dataset of Envisat images. Generated a database with different pairs of Envisat images, in the same location it is very important for investigate the quality of an interferogram and on the same time observation and estimation of atmospheric delay or observation of meteorological data with SAR interferograms.

This system described that for every combination set of interferogram the difference of the unwrapped phase delay between them is as result of fluctuation of the atmospheric effect with the obligation that the parameters of height region is the same. In that case the matrix $\Phi$ has deficient rank because the rank of matrix defines by the linearly dependent rows namely with n, which is, number of interferograms is less then $(n + 2)$ which is number of relationship generally.

In this way this description (1) for different n SAR interferograms embodied in the system (2):

$$\Phi_1 = \Phi_H + \Phi_{at_1} + \Phi_{ERR}$$
$$\Phi_2 = \Phi_H + \Phi_{at_2} + \Phi_{ERR}$$
$$\Phi_3 = \Phi_H + \Phi_{at_3} + \Phi_{ERR}$$

$$\vdots$$
$$\vdots$$

$$\Phi_n = \Phi_H + \Phi_{at_n} + \Phi_{ERR}$$

A linear system of equations is a set of n-linear equation in $k=n+2$ variables. Linear systems can be represented in matrix form as the matrix equation $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$

Can be solved by taking the matrix inverse to obtain: $\mathbf{x} = \mathbf{A}^{-1} \mathbf{b}$

The help of matrices can express each of these systems. We can denote such system shortly as $\Phi \cdot \mathbf{x} = \mathbf{B}$. Then $\mathbf{x}$ is the column matrix of the unknowns. Therefore we can gather all essential data of the system in one matrix, by adding the column matrix of the known terms to the coefficient matrix.

In this condition $k=n+2$ the system is over determined. Our case is specifically because the rank of matrix $\Phi$ is defined by n interferograms in use and the difference (n-k) is constant. We have the low rank ($\Phi$) problem with. In this way we arrive the unique solution of the (2), and the developed algorithm described the method for computes the unknown parameters where one of them is atmospheric phase delay for n interferograms included in system generated, so we define the vector of atmospheric phase delay and the phase delay from the unwrapping procedure errors for every respective pixel in each interferogram.

The solution of rank-deficient for the new system build is the same with the QR-factorization, since the rank of matrix ($\Phi_i$) is deficient because the rank of this matrix is the same with number of interferograms used and the number of variables in these relationships is greater. Solving this system, we can estimate the vector data with atmospheric phase for every master and slave images date respectively.

So this solution is going to help us to study atmospheric observations in different interferograms.

ii) Effect of baseline on phase delay

Using satellite positional imaging parameters we can convert these phase values into elevations via a simple formula [15].

$$h = \Phi \cdot (\lambda r \sin \alpha) / 4\pi B$$

Where $B \perp$ is the perpendicular component of the baseline, $\alpha$ is the incidence angle, $\Phi$ is the phase value of the interferogram, $r$ is the range of the interferogram pair and $h$ is the elevation of ground at the point of observation.

Envisat’s ASAR sensor uses a wavelength of 5.6 cm and hence, interferometry with ASAR allows us to measure differences in distance in the order of centimetres, or even millimetres.

For practical-real applications, the phase $\Phi$ is not only due to the elevation of the target, but also due to atmospheric delay and unwrapped procedure. Thus the phase angle $\Phi$ is the algebraic sum of the phases due to topography, errors procedure and the atmosphere. Therefore (3) can be written as:

$$h = (\Phi_H + \Phi_{ERR} + \Phi_W)C / B$$

Where $C = (\lambda r \sin \alpha) / 4\pi$ ($\lambda \sin(\alpha) / 4\pi$ is an constant which got different values for each interferogram. As much the system (4) is:

$$\begin{align*}
\Phi_1 \cdot C / B_1 &= \Phi_H \cdot C / B_1 + \Phi_{at_1} \cdot C / B_1 + \Phi_{ERR} \cdot C / B_1 \\
\Phi_2 \cdot C / B_2 &= \Phi_H \cdot C / B_2 + \Phi_{at_2} \cdot C / B_2 + \Phi_{ERR} \cdot C / B_2 \\
\Phi_3 \cdot C / B_3 &= \Phi_H \cdot C / B_3 + \Phi_{at_3} \cdot C / B_3 + \Phi_{ERR} \cdot C / B_3 \\
\vdots & \vdots \\
\Phi_n \cdot C / B_n &= \Phi_H \cdot C / B_n + \Phi_{at_n} \cdot C / B_n + \Phi_{ERR} \cdot C / B_n
\end{align*}$$

IV. IMPLEMENTATION AND EVALUATION

The idea of the algorithm is:

a) To find the decomposition matrices Q and R,

b) Solve $Qz = b$ by back-substitution.

c) Solve $Rx = y$ by back-substitution.

In other words the QR-factorization decouples the original problem $Ax = b$ into two problems, one of them ($Qy = b$) has no error application, the other one ($Rx = y$) has the minimal possible error application allowed by the inherent error application of the original problem.

The new method for indirect observations and estimate values of atmosphere in water vapours and the unwrapping procedure errors has been implemented onto ENVISAT imagery of Attica, Greece. It is the goal of this paragraph to go into a detailed description of the results of the evaluation.
The investigation the validation of the SAR signal delay for tropospheric delays can be validated with GPS estimates. In the figure 1 are described the cushion of the result of the developed method with the result that are given with the GPS metrics in four different stations. GPS time series wet delay estimations for these dates will be collected for one station in each study area. These will be compared with the equivalent delay changes obtained from the relative SAR interferograms.

The study of meteorological data (P,T,H) of the dates of captured the images, from the stations in Attica it is very important for the estimated atmospheric delays have been evaluated by the tropospheric delay estimated through interferometric procedure. For this reason it used the database from national metrological service.In the figure 2 are described confront between the Saastamoinen model [16] and the method developed.

Figure 1. Confront of proposed method with GPS data for four different stations in Attica, Greece.

According to the evaluation data, we observe that the method developed successfully and the deviation from the real value for the scarce number of interferogram is admissible.

Figure 2. Confront of proposed method with Saastamoinen model result for four different stations in Attica, Greece.
In all cases we have analytical solution and what they look like is that the estimation method can be assessed for compute the solution of our model.

V. CONCLUSIONS
An approach for calculated on the estimate errors unwrapping procedure and indirect observation atmosphere has been presented here. The estimate for respective optimal values of the described parameters developed based on QR-factorization for solves the system with different interferograms, which simply method. The proposed method will contribute to the operational use of the interferometry of the Envisat SAR images by a) improving the accuracy of the unwrapping procedure and consequently the derived DEM accuracy, b) correlating interferograms to the atmospheric signal delay and, consequently to weather prediction.

REFERENCE