A Differential Evolution Tuned Optimal Guidance Law

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Abstract—The optimal guidance law (OGL) is based on the assumptions of linear kinematical model of missile guidance and unconstrained control in a linear quadratic regulator formulation. These are strong assumptions that are rarely satisfied in practice; the kinematics of the missile-target engagement is highly nonlinear, and infinite lateral acceleration – which is the control - is a physical impossibility. As a consequence, the application of OGL to the actual nonlinear model shows a large miss distance except for a limited range where the linearity assumption is valid and the control needed is not so large as to hit the constraint boundary.

This paper proposes a novel improvisation of the OGL using differential evolution (DE) that compensates the effects of these assumptions and increases the range of validity of OGL. Treating the missile-target kinematics as a black box in which nothing is known except the input and the output, the output is optimized for variations in input using DE. The nominal values of the input variables used for the OGL are used to seed the initial population of trial solutions.

I. INTRODUCTION

The optimal guidance law (OGL) is the optimal control needed to minimize either the miss distance or both the miss distance and the total lateral acceleration, subject to the dynamic constraint in the form of a state equation describing the kinematics of the missile-target engagement [1], [2]. In order to obtain an analytical solution, certain simplifying assumptions are imposed: the state equation constraint is linear, and the control is unconstrained [1]. The solution or control obtained subject to these assumptions is the optimal guidance law (OGL). However, in practice, these assumptions prove to be too strong to reflect the reality. The kinematics involved is known to be highly nonlinear [3], and the control available is limited by the acceleration saturation that every physical system like a missile is subject to. Comparison of the miss distances predicted by applying the OGL to the linearized model of the kinematics and the actual nonlinear kinematic model (hereafter referred to as the ‘plant’) has been investigated [4]. Significant discrepancy between the results for the linear model and the plant has been reported therein, which concluded by expressing the need for a new guidance law that takes into account the effect of nonlinear kinematics.

The present paper proposes a novel improvisation of the OGL that remedies the effects of these assumptions. The parameters used in the OGL are tuned (or adjusted) about the nominal values obtained from the linear model by using the differential evolution (DE). These tuned values of the input variables are then used in the OGL, resulting in a much improved performance. Thus the mismatch between the model assumed for deriving the OGL and the actual plant can be compensated over a larger range about the linearizable conditions.

This paper is organized as follows: Section II gives a brief introduction to the guidance problem from a perspective necessary to develop the paper. The problem formulation and solution methodology is developed in Section III, followed by the implementation and results in section IV, and the concluding remarks in section V.

II. MISSILE GUIDANCE AS AN OPTIMAL CONTROL PROBLEM

Consider the two dimensional or planar point-mass missile-target engagement [1], shown in Fig. 1. Under the assumptions of constant missile and target speeds ($V_M$ and $V_T$ respectively), and constant acceleration magnitude of the target evasion maneuver ($n_T$), the kinematical equations of the missile-target engagement can be concisely expressed in state variable form as

$$
\begin{bmatrix}
\dot{\beta} \\
R_{T1} \\
R_{T2} \\
\dot{V}_{M1} \\
\dot{V}_{M2} \\
\dot{n}_L
\end{bmatrix} =
\begin{bmatrix}
\frac{n_T}{V_T} \\
-V_T \cos \beta \\
V_T \sin \beta \\
-n_L \sin \lambda \\
n_L \cos \lambda \\
V_{M1} \\
V_{M2} \\
(-n_L + n_L) / T
\end{bmatrix}
$$

(1)

where $\lambda = \tan^{-1}(R_{T1}/R_{T2})$, $n_L$ is the actual acceleration that results when acceleration $n_L$ is commanded, and $T$ is the time-constant of the missile guidance system. The subscript ‘1’ denotes the component along axis 1 and the subscript ‘2’ the component along axis 2. For instance, $R_{T1}$ is the component of $\dot{R}_T$ along axis 1, and $\dot{R}_{T2}$ the component of $\dot{R}_T$ along axis 2.
that along axis 2. $R_M = (R_{M1}, R_{M2})$ and $R_T = (R_{T1}, R_{T2})$ are the positions of the missile and the target respectively. The acceleration $n_c$ is the control input to the plant given by (1).

For ease of analysis, (1) is linearized to give the much simpler, linear engagement model [1]. The state model of the linearized system is given by

$$\begin{bmatrix}
\dot{y} \\
\dot{y} \\
\dot{n}_T \\
\dot{n}_L
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1/T
\end{bmatrix}
\begin{bmatrix}
y \\
\dot{y} \\
n_T \\
n_L
\end{bmatrix} +
\begin{bmatrix}
0 \\
-1 \\
0 \\
1/T
\end{bmatrix} n_c \tag{2}
$$

where the relative acceleration $\ddot{y}$ is the difference between the missile acceleration $n_c$ and the target acceleration $n_T$. The actual acceleration produced $n_c$ is assumed to be related to the commanded acceleration $n_t$ by a first order lag, or the transfer function $\frac{n_c}{n_t} = \frac{1}{1+st}$. A double integration yields the relative vertical position $y$, which at the end of the engagement, $t = t_f$, is the miss distance $y(t_f)$. By assuming that the closing velocity $V_c$ is constant, the relative range at any time $t$ during the flight time $t_f$ is given by

$$R_{TM} = V_c t_{go}, \quad \text{where } t_{go} = t_f - t$$

The problem of finding the optimal control input can be formulated as

$$J = \int_0^{t_f} n_c^2 \, dt \tag{3a}$$

subject to the dynamic constraint (2), and

$$y(t) = 0 \quad \text{and } y(t_f) = 0 \tag{3b}$$

where $y(t_f)$ miss distance.

The solution to the above, or the optimal control, is given by

$$n_c = \frac{N'}{t_{go}^2} [y + \dot{y} t_{go} + 0.5 n_t t_{go}^2 - n_t T^2 (e^{-x} - 1 + x)] \tag{4a}$$

where $x = t_{go}/T = (t_f - t)/T \tag{4b}$

and $N' = \frac{6x^2 (e^{-x} - 1 + x)}{2x^3 + 3 + 6x - 6x^2 - 12xe^{-x} - 3e^{-2x}} \tag{4c}$

is the well known Optimal Guidance Law (OGL).

The miss distance $y(t_f)$ can be zero on applying the OGL only if the $n_c$ is unconstrained. If not, (3b) would no longer be satisfied.

III. PROBLEM FORMULATION

Application of the OGL to the linear model (2) shows that the OGL produces very low miss distances as the missile-target separation is varied. In the present paper, the numerical example to which these laws were applied is described in section IV. The initial conditions are of a head-on encounter, with the velocity vectors aligned to the line of sight. In other words, we start with the initial conditions that seem to make a perfect case for linearization. The peak acceleration commanded $n_{c,\text{max}}$, the total acceleration $\int_0^{t_f} |n_c| \, dt$ and the miss distance $y(t_f)$ are all minimum for the OGL. However, these results do not translate to the nonlinear system: when these laws are applied to the actual plant given by (1), it is observed that the miss distance performance of the OGL is much poorer than the well known and widely applied proportional navigation (PN) and augmented proportional navigation (APN) laws, for large $t_f$’s (Fig. 2). Since one reason for this is the approximate nature of the estimated $t_f$, $t_f$ was determined accurately by a separate simulation, and this $t_f$ was supplied as one of the inputs. But it was found that this did not still reduce the miss distance to an acceptable level. The main reason for the poor performance is that the OGL is derived for the linear model, but is applied to the nonlinear plant, outside the range of validity of linearization. This is corroborated by [4], which also explains the reason for this phenomenon: for large $t_f$’s, the small angle approximations made to linearize the nonlinear system are not valid towards the end of the engagement. Another important reason for the large miss distance that we found during our simulations is the unavailability of unconstrained control which is assumed for mathematical tractability while solving (3) by linear quadratic theory. This makes a case for an improved guidance law which will perform as well when applied to the plant. In other words, instead of solving the linear optimal control problem (3), the exact problem to be solved for minimization of miss distance is a Mayer form optimal control problem [5] described below:

$$\text{Minimize } J(n_c(t)) = X^T(t_f) S X(t_f) \tag{5a}$$

subject to (1), that is of the form

$$X = f(X, n_c) \tag{5b}$$

$$|n_c| \leq n_{c,\text{max}} \tag{5c}$$

$$X(t_0) = X_0, \quad X(t_f) \text{ free, } t_f \text{ free} \tag{5d}$$

General closed form solutions are difficult to arrive at for optimal control problems. For Mayer type optimal control problems, this is even more involved, since the Hamiltonian is linear in control input $u$, making the problem singular. A more difficult complication is the presence of the inequality constrained input represented by (5c). The difficulties are enumerated and explained in [6]. The solution may consist of ‘bang-bang’ control, or partly singular, and partly bang-bang control [7].

To overcome the difficulties outlined above, some recent approaches have focused on soft computing techniques, as part of the much larger in scope intelligent control theory. Several specialized software like SOCS, NEOS, NPSOL,
The approach in this paper is to solve the following optimal control problem, instead of (5):

Minimize \[ J = R_{TM}(t_F) \]  

subject to

\[
\begin{align*}
(1) & \quad 0 \leq n_c \leq n_{c, \text{max}}. \\
(6b) & \quad -5 \leq N' \leq 5, \\
(6c) & \quad \frac{T}{\hat{T}} \in [0.8, 1.2], \\
(6d) & \quad \frac{t_F}{T} \in [0.8, 1.2],
\end{align*}
\]

The missile-target kinematics (6b) and the bounds constraints (6c) and (6d) can be treated as a black box, with its input as \( t_F \) and \( T \) and its output as the miss distance \( R_{TM}(t_F) \). The control input \( n_1 = n_1(t_F, T, v, y) \), varies as the input quantities \( t_F \) and \( T \) are varied, in turn varying the output \( R_{TM}(t_F) \) of the black box. The control input is fixed at the bound, if it exceeds any bound in its time history; similarly \( N' \) is fixed at the bound if it exceeds any bound. The minimum \( R_{TM}(t_F) \) can be found by an exhaustive search by varying the input variables over their whole range. However, an intelligent search technique employing directed random search like any evolutionary algorithm can produce a quicker solution. The nominal values of \( \hat{t}_F \) given by (7) and \( \hat{T} \) (specified in the problem data) are used to seed the initial population.

IV. IMPLEMENTATION AND RESULTS

The proposed method is implemented in the following numerical example [1].

\[
\begin{align*}
R_{M1} &= 0, \quad R_{M2} = 10,000 \text{ ft}, \quad R_{T1} = 40,000 \text{ ft} \\
R_{T2} &= 10,000 \text{ ft}, \quad V_M = 3,000 \text{ ft/s}, \quad V_T = 1,000 \text{ ft/s} \\
n_T &= 96.6 \text{ ft/s}^2 = 3 \text{ g}
\end{align*}
\]

The heading error (HE) was assumed to be zero. Although it is possible to achieve a maximum lateral acceleration of more than 30g [2], we wanted to limit the acceleration advantage of the missile over the target to 3:1 to study the performance of our method. Hence, we chose \( n_{c, \text{max}} = 3 \). 

The maximum value of \( N' \) was taken as 5, for the OGL. 

The initial population comprises individuals that are the randomly generated pairs of \( t_F \) and \( T \) about the seed values of \( \hat{t}_F = 1 \) s, and \( \hat{T} = R_{TM}(0)/V_c(0) = 40,000/4,000 = 10 \) s. 

The fitness of each trial solution or individual is evaluated by solving (7d) with the OGL as the control input, and obtaining the miss distance as the output. The best solution is that pair of \( t_F \) and \( T \) which gives a miss distance less than the tolerance during the generations, or the least miss distance after the maximum number of generations allowable.

Differential evolution (DE) was chosen out of all evolutionary algorithms (EAs) available, to speed up the execution, since the DE algorithm is one of the simplest of all EAs. 

The population size used was 12, and the stopping criterion was fixed as the earlier of the tolerance limit of 3 feet of miss distance, or a maximum of 4 generations. This was to avoid the possibly unduly long time delay in running through an unknown number of generations. 

The total acceleration \( \int_{0}^{t_c} |n_c(t)| dt \) could also be added as another term to the performance index. However, this would necessitate the two terms to be weighted suitably, to prevent undue influence of one over the other. Since achieving a suitable balance between these weights is difficult for varying flight times, the total acceleration term has not been added.
since optimal missile guidance is an optimization problem that has to be done online. The coding and simulation was done in Matlab® language. The fourth-order Runge-Kutta method was used to solve (1) with a step size of 0.01 sec. The DE implemented used binomial crossover, with a crossover constant (CR) of 0.9, and weighting factor (F) of 0.8, with parts of the code adapted from [8]. Our simulations showed that, for this particular problem, the variation of CR and F did not make any significant difference to the convergence of the solution. Moreover, there was not a single instance of failure to converge during our simulation runs.

Fig. 2 shows the miss distances obtained for various $t_f$’s, for various initial separations of the missile and target. It can be seen that over the entire range of $t_f$, the proposed DE-OGL gives the least miss distance. Fig. 3 and 4 give an indication of how this is achieved. The proposed method dictates higher values of $n_c$ and $N'$ in the initial stage than the OGL. It is to be noted that the control input $n_c$ is partly singular (the initial part of the flight) and partly bang-bang (from about 6.92s, approx.). The improvement in solutions (miss distance) as the generations proceed is shown in Fig. 5.

V. CONCLUDING COMMENTS

A new differential evolution tuned optimal guidance law (DE-OGL) that performs better than the conventional OGL has been presented. DE was used to overcome the non-trivial restriction that inequality constraints in a nonlinear optimal control problem posed.

The price paid for the improved performance is increase in computational requirement and an increase in the difficulty of realization of the guidance law. Further research in applying evolutionary algorithms for online implementation is under progress, given that missile guidance is a real-time optimization application.

As compared to the method in [7], the approach proposed in the present paper is much simpler, but more restricted in terms of the initial conditions that can be handled. The approach in [7] could start from any initial conditions.

It is to be noted that the solution herein was obtained offline, since the computation time is slightly higher than the time period for which the solution has to be applied. However, since there are only two variables to be tuned, over a limited search space, the method offers the promise of being implementable online. In this respect, it seems a better prospect than the approaches found in [7] and [9], which have used GA for guessing good initial trial solutions alone, and used shooting method thereafter for obtaining the final solution.

As with all heuristic approaches to research, the strongest justification for the proposed method herein is that they work. It is only after some heuristic approach works in practice that researchers try to deepen their understanding of the approach through more experimentation and by means of an effort to build a theory [10]. To quote another reference [7], ‘General convergence arguments pertaining to the GA are mainly heuristic and based on practical experience. Because of the probabilistic nature of GAs, this seems unavoidable.’ Any justification that applies to GA applies to the DE as well.

Stability of the system with the control obtained by the proposed method is another serious issue that needs to be looked into and is not dealt with in this paper.

REFERENCES


Fig. 1. Two-dimensional missile-target engagement model
Fig. 2. Miss distances of the DE-OGL and other guidance laws for the actual plant

Fig. 3. $n_c$ requirement of all laws

Fig. 4. $N'$ requirement of the OGL and DE-OGL

Fig. 5. Improvement of the solution through the generations