Federated filtering for fault tolerant estimation and sensor redundancy management in coupled dynamics distributed systems

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Abstract—This paper discusses the application of the idea of federated filtering to the estimation of intrinsically nonlinear distributed systems by examining its impacts on filtering performance by using the Extended Kalman Filter (EKF) as state estimator. Specifically, the performance of the traditional centralized solution is compared with the filtering structure obtained using the federating idea, and their conceptions, and their ability to balance between fault tolerance and estimation accuracy is examined. Our research demonstrates how successfully the EKF for solving nonlinear estimation problems in federated structures can be used, noting that the idea of federation have only been demonstrated for linear problems previously. In addressing the demands of both fault tolerance and estimation accuracy, it is shown that increased filtering accuracy is relied on the proper choice of sharing of the error covariance information between local filters, while sensor fault tolerance is provided by the utilization of an appropriate resetting policy of the filter.

I. INTRODUCTION

Distributed systems are increasingly important in a widening array of applications in process control, information and communication systems, sensor networks, vehicle technology, just to mention a few. The decentralized implementations do not just benefit from a designs that can provide more cost-effective solutions, but they are required for effective functioning. Advanced vehicle onboard control systems, including land, marine and avionic systems, for instance, are increasingly relied on a highly distributed electronic systems architecture. These architectures might contain a dozen of subsystems decomposing overall system functionality to several sub-system functions, which are then individually controlled and supervised by one or more dedicated Electronic Control Units (ECU’s). The multitude of these ECU’s are typically implemented on embedded platforms and interconnected via heterogeneous computer networks. A fundamental problem in such kind of distributed control structures is to solve detection and estimation problems to supply reliable data for controllers, using scalable algorithms which comply with the stringent performance requirements (cf. real-time execution, complexity, modularity, computational energy, etc.) demanded by embedded applications.

As a matter of fact, the dynamics of these systems are characteristically nonlinear, moreover, the representative application domains of this technology tend to be, more and more frequently, safety critical. This requires development of novel fault tolerant methods and algorithms properly fitting to the distributed character of the application problem, for estimation, and particularly fault detection, that are currently unavailable.

It has obvious advantages in assigning the overall computation burden among local processors, and in fault tolerant capability. A centralized filter suffers from the computation burden and is prone to both sensor and implementation failures and the solution cannot answer the calls of the new distributed architectures. The alternative is to process the information locally at the component or subsystems level using local information about the system dynamics.

This paper addresses the distributed estimation problem in nonlinear systems using decentralized filtering. Previous efforts on distributing filtering structures have concentrated on either decentralized filters on centralized or hierarchical topologies or essentially centralized filters on decentralized topologies based on the work of [1], [2], [3].

These implementations assume a truly decentralized architecture requiring no central facilities. Several decentralized results have been developed to decentralize the filter algorithm, topology, and services through tradeoffs of computation and memory that minimize communication. The result is an optimal, linear filter which needs only to share nodal dimension information between autonomous nodes. Since the Kalman filter has been an excellent means for exploring decentralized system tradeoffs because of its optimality and linearity, the majority of solutions rely on derivation of the linear Kalman filter. But there need data transmission between local processors. In real applications, however, the amount of necessitated data transmission may be very large. One particular distributed filtering structure that provide a solution to this problem is referred to federated filtering that is based

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on the information sharing principle developed in [4], [5] and [6]. Federated filtering is also known to have particularly good fault tolerant capability. These structures consists of several local filters (LF’s) and a master filter (MF). There is a particular LF assigned to each particular subsystem. LF’s work in parallel, and their solutions are periodically fused by the MF. Although, decentralized federated structures have been extensively studied for reducing the typically high computational load of standard (centralized) filtering, its potential to fault tolerance and performance increase has not been investigated and fully realized. Moreover, whilst the techniques and theory of federated filtering (especially those based on the Kalman filter) are relatively well developed for linear systems, experiences subjecting nonlinear applications are not yet widely available.

In addressing the problems of both fault tolerance and estimation accuracy, in this paper the applicability of the extended federated filtering idea which uses the Extended Kalman Filter (EKF) as state estimator and a sensor redundancy management logic to maintain estimation functionality in nonlinear systems is investigated. EKF has become a standard technique of estimation in systems where the state and observation models are not linear functions of the state but instead they are general (differentiable) functions. EKF essentially linearizes the non-linear function around the current estimate by using standard numerical techniques. The approximation issues are sometimes treated with using polynomial approximation techniques see, [7].

The state estimation problem of a special class of systems is considered, namely, distributed systems which can be decomposed of the set of subsystems having nonlinear coupled dynamics. By coupled dynamics the coupling or mutual dependence between the elements of the global system state is meant. Apparently, the federated state estimator solutions will be different providing this assumption is considered valid or not.

The coupling has several implications on the filtering solution. The most important consequence is that each LF should provide an estimate of the global state vector based on a set of subsystem specific local measurements for best accuracy. Therefore, the local filters are not smaller dimensions than that of the centralized one in this case, but performance gains and increased robustness makes the application of this special filter architecture highly favorable in the practice.

Another consequence of the coupling is that estimation of a particular state of the global state vector can be achieved based on several different redundant measurements. Estimations based on rearranged sensor layout in a centralized architecture may provide considerably different estimation performance. Sensor assignment, therefore, is one of the main concerns of the design in distributed systems having coupled dynamics. As a related problem of sensor redundancy, reliability of individual sensors is frequently inadequate to satisfy the stringent reliability requirements in complex safety critical systems. Therefore, an array of redundant sensors is usually employed to achieve the required system reliability through the use of redundancy. Traditionally, this idea relies on the utilization of a sensor failure detection and isolation system to detect sensor malfunctions. It is shown in this paper, how federated solutions exploit the principle of redundancy to improve both estimation accuracy and fault tolerance in the same time.

The paper is organized as follows: in Section 2, the basic features of the federating solution is briefly given. The necessary theory of the design of the federated filter is focused on the presentation of the fusion algorithm, the discussion of the EKF adopted for solving the nonlinear estimation problem is not detailed. We take the classical federated filter idea one step further by adding fault detection and sensor management to the architecture. A simulation example, demonstrating and comparing the features and performance characteristics of various solution alternatives is presented in Section 3.

II. FEDERATED FILTER ARCHITECTURE

Federated filtering is usually regarded as a two-stage data processing technique. In the structure of filters composed of the set of local and the master filters, the LF’s work in parallel, independently of one another, and their solutions are periodically fused by the MF yielding a global solution. For the solution of the nonlinear state estimation problem, in this paper the nonlinear extension of the well-known linear Kalman filtering algorithm is used in a particular structure, where both local and the master filters adopt the EKF idea for filter implementation. As both Kalman filter and the EKF considered to be standard techniques of control theory by now, only the details necessary to the discussion are mentioned here. For more information, the interested reader is referred to the literature, such as e.g., [8] and [9].

In this work we are concerned with nonlinear dynamical systems in the nominal representation described by ordinary differential equations subject to noise

$$\begin{align*}
\dot{x}(t) &= f(x(t), u(t), w(t)), \\
y(t) &= h(x(t), v(t))
\end{align*}$$

which can be written in state space form, by means of a set of equations of the following type

$$\begin{align*}
\dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)u_i + w, \\
y_i &= h_i(x) + v_i, \quad 1 \leq i \leq p,
\end{align*}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ denote respectively the state, the input and the output of the system, $w(t)$ and $v_i(t)$ are the system and the measurement noise, respectively. Assume each sensor $s_i$ is measuring a signal $y_i(t)$ that is corrupted by measurement noise $v_i$ that is a zero-mean white noise. Let $Q_i$ and $R_i$ denote the covariance matrix of $v_i$ and $w$ for all $i$, respectively

$$Q = \mathbb{E}\{w_k w_k^T\}, \quad R = \mathbb{E}\{v_k v_k^T\}. \quad (3)$$

Let the local estimate and its covariance provided by the $i^{th}$ local filter be represented by $\hat{x}_i$ and $P_i$ (i =
1, 2, ..., N), as is shown in Fig. 1. The filtering algorithm is considered the extension of the standard linear one. Since the system is not linear the Riccati matrices that attempt to approximate the a priori and a posteriori covariances for each filter are defined, respectively, as

\[ P_{k|k-1} \approx \mathbb{E}\{ e_{k|k-1} e_{k|k-1}^T \}, \]  
\[ P_{k|k} \approx \mathbb{E}\{ e_{k|k} e_{k|k}^T \}. \]

The filters are initialized with \( \hat{x}_{0|0} = x_0 \) and \( P_{0|0} = P_0 \), and then they operate recursively performing a single cycle each time a new set of measurements becomes available. Each iteration propagates the estimate from the time the last measurement was obtained to the current time. The propagation process consists of two stages: update and prediction. The update equations are responsible for the feedback, i.e., for incorporating a new measurement set into the a priori estimate to obtain an improved a posteriori estimate. The a posteriori state estimate \( \hat{x}_{k|k} \) is computed as a linear combination of an a priori estimate \( \hat{x}_{k|k-1} \) and a weighted difference between an actual measurement \( y_k \) and a measurement prediction:

\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left( y_k - h(\hat{x}_{k|k-1}) \right), \]  
\[ K_k = P_{k|k-1} \tilde{H}_k (H_k P_{k|k-1} \tilde{H}_k^T + R_k)^{-1}, \]  
\[ \tilde{H}_k = \left( \frac{\partial h(x)}{\partial x} \right)_{x=\hat{x}_{k|k-1}} \]

The matrix \( K_k \) is chosen such that the filter minimizes the a posteriori error covariance. The covariance matrix is updated by

\[ P_{k|k} = (I - K_k \tilde{H}_k) P_{k|k-1}. \]

The predictions equations are responsible for projecting forward, in time, the current state vector and error covariance estimates to obtain a priori estimates for the next time step. The state and covariance matrix in the next sampling instant are estimated by

\[ \hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k), \]  
\[ P_{k|k+1} = \tilde{F}_k P_{k|k} \tilde{F}_k^T + Q_k \]

The following characteristics distinguish the LF’s and the MF. Each LF is dedicated to the measurements of local sensors. For the update process, the LF’s use the update information from its local sensors only and not any others. Conversely, the MF uses the local filtered estimates \( \hat{x}_i \) as quasi-observables to update the global state vector in a fusion process, sequentially. The fused estimation of the MF is usually based on measurements \( y_R \) provided by reference sensors. Reference sensors act as fundamental sensors in the system, which are usually more dependable and have higher data rate; thus their data is often used for the initialization of some of the local filters and accountable to compensate erroneous sensor measurements relying on the decentralized architecture as will be shown later.

The MF is processed at the rate equal to the rate of the LF’s which means that local outputs are subject to fusion in the next stage they are processed. The time updating solution of the MF is represented by the state estimation \( \bar{x}_{N+1} \) and covariance \( \bar{P}_{N+1} \), respectively. If all local estimates are uncorrelated, then the global estimate can be given as

\[ P_f^{-1} = P_1^{-1} + P_2^{-1} + ... + P_N^{-1} + P_{N+1}^{-1}, \]  
\[ \bar{x}_f = P_f^{-1} \bar{x}_1 + ... + P_N^{-1} \bar{x}_N + P_{N+1}^{-1} \bar{x}_{N+1}, \]

where the inverse of the covariance matrix is called the information matrix. Eq. (11) shows that the global information is just the sum of that of the local systems. The global estimate \( \bar{x}_f \) is a linear weighted combination of the local estimates with weighting matrices \( P_f^{-1}, P_i^{-1} \) \( i = 1, \ldots, N, N+1 \). However, in fact the estimates of different local filters are correlated because of our assumption on the coupled dynamics. In order to eliminate this correlation, the process noise and state error covariance are set to their upper bounds as follows:

\[ Q_i = \beta_i^{-1} Q, \quad P_i = \beta_i^{-1} P_f, \]

where \( \beta_i \geq 0 \) is the information-sharing factor satisfying \( \beta_1 + \beta_2 + \ldots + \beta_N + \beta_{N+1} = 1 \). For this condition that makes Eq. (11) holds, see [6]. This structure of filters can be operated in basically two different operating modes depending on how the fused process data is exploited by the local filters. When the reset mode is used, the master an local filters are reset by the global solution \( \bar{x}_i = \bar{x}_f \) and \( P_i = \beta_i P_f \), for \( i = 1, \ldots, N, N+1 \). That is to say there is a continuous information feedback from master filter to local filters. In no-reset mode, however, this information feedback does not exist: each LF keeps its process information \( (\bar{x}_i, P_i) \) produced locally, thus the MF retains none of the fused data and the global fused estimation \( (\bar{x}_f, P_f) \) has no effect on any of the local estimations. There are obvious advantages and disadvantages
associated with each of the resetting modes. The federated filter operated in reset mode is expected to provide better estimation accuracy, while in the no-reset mode a better tolerance of sensor faults.

III. APPLICATION

A. Objectives

Sensor failures causing measurement drop-outs, sensor noise and measurement uncertainty will impose a practical limitation on the estimation accuracy obviously. However, the application of the federated filtering idea provides a variety of possibilities for improving filter performance. In the remainder of this paper the effect of the selection of the operating mode is investigated, based on the application example presented in the next section. The classical federated architecture is extended with sensor fault detection and a sensor management logic, in an attempt to make a tradeoff between best estimation accuracy and sensor fault tolerance in every possible time. The key to this solution is to determine how total information generated by the local filters is to be divided among the individual filters by proper assignment of the information sharing factors $\beta_i$ and reset mode in the fusion filter to achieve the best possible estimation accuracy, or if a failure occurs, for higher fault tolerance.

Sensor failures may be caused by sensor drift, step changes, scale factor errors changing the mean, incorrect calibration of the sensors, etc. The degraded modes of sensors are characterized by the presence of a systematic nonzero mean, in the form of a constant jump bias, a ramp bias, or by an increase in variance of the driving noise. The diagnosis approaches can be based on the analysis of the innovation sequences of the filters. To detect failures changing the mean of the innovation sequence a number of statistical approaches are available that make use a changing the mean of the innovation sequence a number of statistical approaches are available that make use of such equipments represent an engineering challenge. Chemical distillation systems are among the most complex nonlinear processes. Both control and estimation of such equipments represent an engineering challenge which requires the application of sophisticated methods of advanced nonlinear control theory. Some of these systems, for instance, are controlled by Model Predictive Control (MPC) that necessitates the availability of accurate state estimation. The process presented here consists of 30 separate distillation stages, i.e., column trays, identified by

the parameter $N_T$, plus the reboiler and a total condenser as usual. The control-input is the reflux of liquid flow rate which acts on the plant. The sensor outputs used for control and estimation purposes are the temperatures of the column tray. It is assumed that there is one temperature measurement available for each tray. Due to the coupled nature of the nonlinear dynamics, these sensors provide a highly redundant set of measurements upon which the estimation and control of the process can be based. The trays (and measurements) are numbered from the top to the bottom of the column. The instrumentation includes temperature sensors of different quality. There are two highly reliable reference sensor used, one at the topmost and one at the bottommost of the column. The nonlinear model of the distillation column can be represented by the following set of equipment models. The main assumptions and conditions adopted for the model development are not mentioned here, they can be found in the literature. According to [11] e.g., the condenser is subject to

$$M_1 \frac{dx_{i,1}}{dt} = V_s y_{i,1} - L_s x_{i,1} \left(1 + \frac{1}{R}\right)$$

with reflux ratio $R$. A generic tray in the enriching section (rectifying) is modeled as

$$M_j \frac{dx_{i,j}}{dt} = V_s y_{i,j+1} + L_s x_{i,j-1} - V_s y_{i,j} - L_s x_{i,j},$$

for $i = 1, ..., N_c - 1$ and $j = 1, ..., f - 1$ where $f$ is tray where the column is fed. Then the feed tray is subject to

$$M_j \frac{dx_{i,j}}{dt} = V_s y_{i,j+1} + L_s x_{i,j-1} - V_s y_{i,j} - L_s x_{i,j} + F_j z_{i,j},$$

with feed flow rate $F$ and molar composition $z_i$ of component $i$. A generic tray in the stripping section is represented by

$$M_j \frac{dx_{i,j}}{dt} = V_r y_{i,j+1} + L_r x_{i,j-1} - V_r y_{i,j} - L_r x_{i,j},$$

for $j = f + 1, ..., N_T - 1$, and the reboiler is modeled as

$$M_{N_T} \frac{dx_{i,N_T}}{dt} = L_r x_{i,N_T-1} - V_r y_{i,N_T} - L_r x_{i,N_T}.$$  

(18)

In (13)-(17) $N_c$ stands for the number of liquid components, $M_j$ is the molar holdup corresponding to a particular tray, $y_{i,j}$ is the vapor composition of component $i$ at stage $j$, and $V_r$, $L_r$, moreover, $V_c$ and $V_s$ are the liquid and vapor flow rates of the stripping and rectifying sections, respectively. The state variables are the liquid compositions at the trays. For a system with NC components ($N_c = 3$ in this case), $N_c - 1$ state variables are considered in every tray. Thus, there are $64$ state variables included in the global model. The composition of the $N_c$th component is obtained by

$$x_{i,N_{c,j}} = 1 - \sum_{i=1}^{N_c-1} x_{i,j},$$

$1 \leq j \leq N_T$. 

(18)
The temperature on each tray is obtained using the Vapor Liquid Equilibrium equations, see [12]. The model equations (13-17) are subject to implementation for the realization of the EKF filters (both locals and master). The simplifications included in the modeling process introduce modeling errors for which the individual EKF's are expected to compensate.

C. Simulation results

In this section, we present the results obtained for the solution of the nonlinear state estimation problem using various filter layouts and process conditions. The process simulations were performed in Matlab using the KALMTOOL package for the EKF implementations, see Ref. [7]. A federated filtering scheme, resembling to the structure depicted by Fig. 1, containing three local and one master filter was derived for evaluation.

For state estimation, observability of the nonlinear system representation is required. As it was shown in Ref. [13], the ternary mixture distillation process is observable if at least two temperature measurements of the column are available. The performance of the estimator may improve if more than the minimum necessary measurements are used. We present the case when the minimum number of measurements is used, i.e., only two of the 32 temperature measurements are available in the column for the synthesis of the filters. Another relevant aspect of the filter design is the proper selection of the measurement locations. Not going into details of sensor assignment, in our case two reference temperature measurements were allocated in the structure: one to the bottom of the column, which has the largest inertia of the system, and the other to the top of the column, so as to reflect promptly changes in the end-product composition. Relying on the above considerations the local filter estimates are based on the tray temperature measurements in the grouping LF1: \((y_{11}, y_{31})\), LF2: \((y_{22}, y_{32})\) and LF3: \((y_{33}, y_{31})\), whilst the master uses the reference inputs MF: \((y_{11}, y_{12})\). In normal operating conditions the normal and reference measurements are assumed to have sensor noise with variance \(\sigma = 0.01\) and \(\sigma = 0.001\), respectively, indicating the higher dependability of the reference sensors.

In simulation of the complex column behaviors, a rigorous distillation model, different from those of (13-17) based on Differential Algebraic Equations (DAE) was employed. The process simulation was based on the assumption that a step change in the control signal (reflux) in zero simulation time is occurred. As an effect, the process imposes a transient behavior where the resulted variation is the subject of the state estimation. Since the composition at the top of the column is in direct relationship with the end product of the distillation i.e., methanol, the estimation of this composition is of the utmost importance and our filtering results are related to this state variable.

Two different types of sensor failures were considered. A sensor failure (shift) changing the mean value of the innovation sequence of the second measurement channel by a constant bias \((\mu = 1)\), moreover increased variance in the driving noise \(\sigma = 10\) instead of the normal \(\sigma = 0.01\) is considered. For more accurate comparison of the results, the same noise sequences were used for each simulation experiments. Experiences for the comparison of the centralized filter performance and the federated idea were carried out. If the system operates normally, the normalized innovation sequence in a filter is a white noise sequence with a zero mean and with unit covariance matrix. A real-time detection of sensor failures affecting the mean and/or the variance of the innovation process triggers a filter reconfiguration action in the sensor management logic of the fusion filter in an attempt to keep the performance of the impaired structure as close to the optimum as possible. This includes the modification of both the sharing factors and the reset mode of the filter. A comparative summary of the results can be found in Table I showing filter performance data corresponding to particular filter configurations and various process conditions. The performance of individual solution alternatives is characterized in terms of the \(L_2\) norm of the estimation error \(\hat{x}\), moreover, with the relative estimation error \(\delta\) given with respect to the estimation accuracy obtained with the centralized filter in fault free case. Simulation results (see Table I) confirm that the centralized and federated filtering operated in full reset mode with identical information sharing factors provide the same performance both in faulty situation and in fault free case. This result was expected, it comes from the federating theory. Differences can be experienced in faulty situations when, after detecting a fault, one switches the federating scheme to no reset mode by masking out the subjected local filter with setting the corresponding \(\beta\) to zero. Fig. 2-7 show the plots of some of the distinguished experiences.

<table>
<thead>
<tr>
<th>Filter Layout</th>
<th>Sensor Fault</th>
<th>(\beta)</th>
<th>(|\hat{x}|_2)</th>
<th>(\delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized</td>
<td>no fault</td>
<td>–</td>
<td>0.0134</td>
<td>0</td>
</tr>
<tr>
<td>Centralized</td>
<td>(\sigma = 10, \mu = 0)</td>
<td>–</td>
<td>0.0192</td>
<td>43.2</td>
</tr>
<tr>
<td>Federated with reset no fault</td>
<td>(\sigma = 10, \mu = 0)</td>
<td>equal</td>
<td>0.0135</td>
<td>0</td>
</tr>
<tr>
<td>Federated with reset</td>
<td>(\sigma = 10, \mu = 0)</td>
<td>equal</td>
<td>0.0192</td>
<td>43.2</td>
</tr>
<tr>
<td>Federated no reset</td>
<td>(\sigma = 10, \mu = 0)</td>
<td>equal</td>
<td>0.0136</td>
<td>1.5</td>
</tr>
<tr>
<td>Federated no reset</td>
<td>(\sigma = 10, \mu = 0)</td>
<td>equal</td>
<td>0.0291</td>
<td>117.2</td>
</tr>
<tr>
<td>Federated no reset</td>
<td>(\sigma = 10, \mu = 0)</td>
<td>(\beta_2 = 0)</td>
<td>0.0160</td>
<td>19.4</td>
</tr>
<tr>
<td>Federated no reset</td>
<td>(\sigma = 10, \mu = 1)</td>
<td>equal</td>
<td>0.0223</td>
<td>66.4</td>
</tr>
<tr>
<td>Federated no reset</td>
<td>(\sigma = 10, \mu = 1)</td>
<td>(\beta_2 = 0)</td>
<td>0.0145</td>
<td>8.2</td>
</tr>
</tbody>
</table>

![Fig. 2. Centralized estimation without sensor fault.](image-url)
The basic design technique of federated filtering consists of using dedicated local filters to particular components of large distributed dynamical systems and utilizing a master filter for obtaining the global estimation. The numerical simulation example presented have shown that the proper selection of the sharing factors, as well as the choice of the resetting policy of the filter may positively affect filtering performance. This support the hypothesis that a better estimation accuracy and sensor fault tolerance in the federated solution in case of sensor degradation can be obtained. The key to this solution is the restructurable architecture represented by the federated idea, as well as a reconfiguration scheme that, with the aid of the utilization of the sensor fault detection and fault management logic, alters the fusion filter law to achieve a better performance. Each filter within a federated group is assigned, under the control of the fault management function of the master filter, to operate in either a full-reset or a no-reset mode. Failure of any of the input channels can be tolerated without significant deterioration of estimation performance. In case the estimation is used in a subsequent control action, without loss of operational capability. This property of the filter confirms the applicability of the idea in dependable distributed control systems.

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