Multivariable PID Control Design
For Wastewater Systems

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Abstract—This paper investigates the application of multivariable PID controllers to a wastewater treatment process. Four multivariable PID control schemes are investigated. The methods considered are those suggested by Davison, Penttinen-Koivo, Maciejowski and a new method proposed in this paper. All of the methods are suitable for multi-input multi-output (MIMO) control loops that experience loop interaction. The methods, furthermore, require only simplistic plant models. The performance of the methods is assessed using a nonlinear benchmark model, and the optimal tuning values are determined using an optimisation method. The simulation results show the significance of the study and lead to the conclusion that the proposed method yields somewhat better results than other methods with respect to decoupling capabilities and closed-loop performance.

Keywords: Multivariable PID, Wastewater Systems, Optimisation, controller tuning

I. INTRODUCTION

Closed-loop control plays an important role in maintaining low operating costs and adequate effluent water quality for Wastewater Treatment Plants (WTP). Traditionally, scalar PID controllers have been extensively used to control the process variables of WTPs [1]. However, due to the inherently multivariable nature of WTPs, combined with an increasing demand for a more consistent effluent water quality, scalar PID-based control systems are often no longer sufficient. To meet the current and future demands on effluent water quality multivariable control systems are therefore needed.

Existing multivariable control techniques are typically model-based and require a significant effort and skill to tune. Although the basic process of wastewater treatment is similar in most plants, their configuration and input characteristics are different. Therefore, a generic model cannot be used for applications such as model-based control. This combined with the fact that industry acceptance for multivariable control techniques generally are low; render the application of multivariable control techniques cumbersome within the WT industry.

In an attempt to improve the WT industry acceptance of multivariable control techniques, this paper investigates the application of four different and essentially model-free, multivariable PID control methods to a benchmark model of a WTP.

The MIMO-PID methods investigated in this paper are those, which require only simple step or frequency response tests to configure and tune. The methods considered are those suggested by Davison [2], Penttinen-Koivo [3], Maciejowski [4] and a new method proposed in this paper. Common for all of these methods is that the information needed for controller tuning requires only plant step-tests and/or the determination of the plant frequency response at a single frequency.

The Davison method introduces decoupling at low frequencies by a constant gain compensator, which is the inverse of the plant model at zero frequency. For stable and square linear systems, the method guarantees asymptotic stability and asymptotic tracking for a particular form of disturbances. The Penttinen-Koivo decouples the plant at both low and high frequencies. In Maciejowski’s method, the plant is diagonalised at a particular bandwidth frequency to minimize the interaction around the system bandwidth. Based on these existing techniques, a new method is proposed to improve control performance while retaining the simplicity of the multiloop strategy in which favour the relevant features required for control of WTPs.

The model used in the study is based on WTP simulation benchmark model developed by the COST Action 624 & 682 Research Group. The validated nonlinear model is based on IWA ASM1 model and the implementation is carried out in Matlab/Simulink [5].

The paper is organized as follows. In Section II the benchmark model used for assessing the suitability of the different MIMO-PIDs is presented. Linearised models of the benchmark system, which later are used for control design, are derived in Section III using subspace identification techniques. The four different multivariable PID control methods are then described in Section IV. Section V describes the selection of the tuning parameter associated with the MIMO-PIDs. The performance of the MIMO-PIDs is thereafter assessed in Section VI. Finally, in section VII, conclusions are drawn from the results of the benchmark study.

II. THE BENCHMARK PLANT

The benchmark plant is comprised of five fully mixed tanks, connected in a series manner as shown in Fig. 1. In each of the tanks a biological reaction is taking place that
helps to degrade the biodegradable matter of the wastewater. Thus, as the wastewater influent flow is passed through the series of tanks its cleanliness progressively increases. The effluent from the tank system is connected to a clarifier, which is finalizes the cleaning process of the waste water (see Fig. 1).

The biological processes that are taking place in the tanks are modelled by the IAWQ Activated Sludge Model No 1 (ASM1) [6], whilst the model used for the clarifier follows (Takács et al.,1991).[7]

In a wastewater treatment system of the type shown in Fig. 1, the biodegradable matter of the wastewater is progressively degraded in the tank reactor system by microorganisms that consume dissolved oxygen (DO). The process of degrading the biodegradable matter can be divided into two stages: nitrification and denitrification. A sufficiently high concentration of DO is needed to satisfy the nitrification process in the aerobic tanks, while a too high DO level will unfavourably affect the denitrification process in the anoxic tanks. A control scheme that can adequately maintain the balance of the DO levels in the system during set point manoeuvring and during influence from external disturbances is consequently essential for an efficient and effective cleaning of the wastewater.

![Fig. 1. Layout of benchmark plant](image)

### III. LINEAR MODELS IDENTIFICATION

The controller design techniques described in this paper can be successfully applied in practice using only very simplistic linear process models of the plant. The simplicity of these models means that they typically can be derived directly from data obtained from plant step-tests. More detailed linear models, if they are available, can also be used, and will often improve the control design by providing additional insight into the dynamic behaviour of the process.

The investigations carried out in this paper employs linear models of the WTP process for control design. The motivation for using linear models in this instance is to gain additional insight into the dynamic behaviour of the WTP process and to allow for a more precise determination of the best controller tuning parameters for each of the control techniques investigated, where the latter will subsequently enable a more objective comparison of the control techniques.

The linear process models of the WTP process were obtained using subspace identification techniques. The use of these techniques is almost as straightforward as carrying out step or frequency response tests. The algorithm employed was N4SID [8], which exhibit robust numerical properties and relatively low computational complexity. Perturbing the benchmark plant inputs using PRBS signals, whilst recording the response on the plant outputs generated data for model identification. During the identification experiments the amplitude and frequencies of excitation signals were selected such as to maximise the information within the bandwidth of each reactor.

The plant outputs of interest were the DO concentrations in aerated tank 3, 4 and 5. The manipulated inputs of interest were the three air flow rates ($K_{va}$) to the tanks. Disturbances, in the form of variations of the influent flow rate and influent ammonium concentration, were also considered. The identified model in dry weather conditions was simulated and the studied controller was applied to nonlinear model. The models obtained from the identification experiments were in discrete form which then converts to the following structure:

\begin{align}
\dot{x}(t) &= Fx(t) + Gu(t) + G_d d(t) \\
y(t) &= Cx(t)
\end{align}

where $y(t)$ is the output vector, $u(t)$ is the input vector, $d(t)$ the measurable disturbance vector and $x(t)$ is a state vector. $F$, $G_u$ and $C$ are matrices of appropriate dimensions. The system transfer function is defined as:

$$G(s) = C(sI - F)^{-1}G_p$$

The identified model given in discrete form was open-loop stable as for a given poles ($0.7703 \pm 0.0153$, $0.9987$, $0.9699$ ) and also controllable and observable. The resulting model had 4 states. To study the loop interactions, the steady state Relative Gain Array, $RGA(0)$. Bristol (1996)[9] was calculated as follows:

$$G(s) = C(sI - F)^{-1}G_p \Rightarrow G(0) = -CF^{-1}G_p$$

$$RGA(0) = \left(-CF^{-1}G_p\right)\circ\left(-CF^{-1}G_p\right)^T$$

where $\circ$ indicates the element-by-element product.

For the studied influent disturbance, the steady state $RGA$ was calculated as:

$$RGA(0) = \begin{bmatrix} 1.5081 & -0.1045 & -0.4009 \\ 0.5604 & 2.0764 & -1.6401 \\ -1.0683 & -0.9744 & 3.0409 \end{bmatrix}$$

Negative off diagonal elements indicate that the corresponding variables should be paired along the diagonal elements. Hence, the DOs in the tanks are controlled by their airflow rates as expected. Big values in the steady state $RGA$ entries indicate strong interactions between the loops.
Loops (2, 3) have the strongest interactions followed by loops (3, 1) and (2, 1).

Since the controller design methods investigated in this paper employs decoupling at specific frequency points, it is useful to examine the dynamic RGA and use the resulting information to decouple the system at frequency points with highest interactions. The DRGA is defined as:

$$DRGA(s) = G(s) \cdot G(s)^T$$  

(7)

The frequency plots both for magnitude and phase of the DRGA for the dry influent rate is shown in Fig. 2-a and 2-b, respectively:

![Fig 2-a DRGA gains for dry influent rate](image)

The DRGA demonstrates that the interactions occur mainly at frequencies about a decade below the open loop bandwidth. The low frequency decoupling is therefore most likely to decentralise the control system and minimised the effect of interactions.

IV. CONTROLLER DESIGN STRATEGIES

The control structures and tuning methods associated with the control techniques investigated in this paper are briefly described below.

**Davison Method:**

The multivariable PID design method suggested by Davison uses only integral action. The control law is given by:

$$u(s) = K_i \frac{1}{s} e(s),$$  

(8)

where $K_i = \varepsilon G^{-1}(0)$ is the integral feedback gain, $G(0)$ is zero frequency gain of the open loop transfer function matrix, $G(s)$, and where $e(s)$ denote the control error. The scalar parameter, $\varepsilon$, is the controller’s single tuning parameter.

Since the integral feedback proportional to the inverse of the plant dynamics at zero frequency, the Davison method is expected to provide good decoupling characteristics at low frequencies.

![Fig 2-b DRGA phase for dry influent rate](image)

**Penttinen – Koivo Method**

The design method proposed by Penttinen and Koivo is slightly more advanced than the Davison method. A proportional term has been added to the control law, giving:

$$u(s) = K_c \cdot e(s) + K_i \frac{1}{s} e(s)$$  

(9)

where, $K_c = \rho [CG_p]^{-1}$ and $K_i = \varepsilon G^{-1}(0)$. The controllers proposed by Davison Penttinen-Koivo are similar in the sense that the integral feedback gains of both controllers are linearly related to the inverse of the plant dynamics at zero frequency, and both controllers are therefore expected to provide good control-loop decoupling characteristics at low frequencies.

Unlike the Davison controller, the Penttinen-Koivo controller also includes proportional control action, where the feedback gain is linearly related to the inverse of the plant dynamics at high frequencies. Therefore, by following the same line of reasoning as above the Penttinen-Koivo controller is expected to exhibit good decoupling characteristics also at high frequencies.

The term $CG_p$ corresponds to the initial degree of slope at each output in response to a unit step input:

$$CG_p = \begin{bmatrix} \hat{y}_{1,1} & \cdots & \hat{y}_{1,m} \\ \vdots & \ddots & \vdots \\ \hat{y}_{m,1} & \cdots & \hat{y}_{m,m} \end{bmatrix}$$  

(10)
where \( m \) is the number of plant inputs and outputs and \( Y_{i,j} \) is the degree of slope at output, \( i \), in response to a step input at input, \( j \).

That the product \( CG \) indeed corresponds to the inverse of the plant dynamics at high frequencies can be shown by applying a Laurent series expansion of the transfer function, eq. 3:

\[
G(s) = \frac{CG_p}{s} + \frac{CFG_p}{s^2} + \frac{CF^2G_p}{s^3} + \ldots
\]

Therefore at high frequencies, the \( K_i/s \) terms are negligible compared to \( K \), so that \( G(s) = CG/\alpha \) and \( G(s)K = I/s \), thus giving the following closed-loop transfer function:

\[
(I + GK)^{-1}GK = \begin{bmatrix} H_1(s) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_m(s) \end{bmatrix} \text{ for large } s
\] (12)

The controller has two scalar tuning parameters, \( \rho \) and \( \epsilon \), which respectively determine the controller’s proportional and integral action.

**Maciejowski Method**

The PID design method proposed by Maciejowski builds on the work carried out by Penttinen and Koivo. However, in Maciejowski’s controller the proportional and integral feedback gains remain equal and linearly related to the inverse of the plant dynamics at a particular design frequency, \( w_b \) \( K_c = \rho G^{-1}(jw_b) \) and \( K_i = \alpha G^{-1}(jw_b) \). The controller tuning parameters \( \rho \) and \( \epsilon \) are scalars.

Since the evaluation of \( G^{-1}(jw_b) \) typically will yield a complex matrix, it is in practice often necessary to employ a real approximation of \( G^{-1}(jw_b) \). This can be achieved by solving the following optimisation problem:

\[
J(K, \Theta) = \|G(jw_b)K - e^{j\Theta T}G(jw_b)K - e^{-j\Theta T}\|^2
\]

\[
\Theta = \text{diag}(\Theta_1, \ldots, \Theta_m)
\] (13)

By appropriately selecting the matrix \( K \) (such that it minimises the above optimisation problem) the product of \( G(jw_b)K \) will be as close to the identity matrix as possible at the design frequency, and therefore provide good control-loop decoupling characteristics around this frequency.

**The Proposed method**

Maciejowski’s control design technique has many tractable properties and an intuitive control structure. Initial benchmark results also indicated that the controller was very effective for the control problem posed by the WTP benchmark problem. However, since Maciejowski’s control design technique involves plant frequency analysis experiments (to obtain the process model) it is predicted that industry acceptance for the technique will be low. This paper therefore proposes a new control design technique that retains some of the properties that makes the Maciejowski controller tractable, but eliminates the need frequency analysis. The proposed control design technique assumes the following control structure:

\[
u(s) = \rho K e(s) + \epsilon K \frac{1}{s} e(s)
\] (14)

where, \( K = [\alpha G(0)(1 - \alpha) CG_p]^{-1} \)

The proportional and integral feedback gain of the proposed controller is a blend between the inverse of the plant dynamics at zero frequency and the inverse of the plant dynamics at high frequency. Thus, provided the plant has low-pass frequency characteristics, a good approximation of \( G^{-1}(jw_b) \) can be obtained by appropriately selecting the additional controller tuning parameter, \( \alpha \).

**V. SELECTION OF THE TUNING CONSTANTS**

To allow for an objective comparison of the performance achieved by the multivariable controllers investigated in this paper the tuning parameters of each of the controllers has been selected such that the following penalty function was minimised:

\[
J = \int_0^{\infty} \dot{x}(t)^T Q \dot{x}(t) + \dot{u}(t)^T R \dot{u}(t) dt
\] (16)

The weighting matrices, \( Q \) and \( R \), were non-negative definite symmetric matrices, tuned such that adequate closed loop performance was obtained.

It was assumed that the process dynamics and controller states could be described using,

\[
\dot{x}(t) = Ax(t) + Bu(t)
\] (17)

\[
y(t) = H\dot{x}(t)
\] (18)

where \( \dot{x}(t) \) and \( v(t) \) denoted the controller integrator states. Under these assumptions the multivariable PID control laws could be expressed using:

\[
u(t) = K\dot{x}(t)
\] (19)

\[
\dot{u}(t) = K\dot{x}(t) = K(A\dot{x}(t) + Bu(t))
\] (20)

where \( K = [K_c, K_i] \). Then, by substituting eq.(20) in eq.(16) the following was obtained:

\[
J = \int_0^{\infty} \dot{x}(t)^T Q \dot{x}(t) + (\dot{x}(t))^T K^T R K (\dot{x}(t))
\] (21)

Then by letting,

\[
Q + (A - BK)^T K^T R K (A - BK) = M
\] (22)

\[
(F - BK)^T P + P(F - BK)
\]

The penalty function could be written as,
The optimal selection of controller tuning parameters for the combined method (the proposed method) was found to be, $\rho = 2$, $\varepsilon = 8669.4$ and $\alpha = 0.995$. For $\alpha = 0.995$, the controller gain matrix is given by:

$$K = \begin{bmatrix} 0.0506 & 0.0119 & 0.0520 \\ 0.0227 & 0.0604 & 0.1040 \\ 0.0278 & 0.0175 & 0.1264 \end{bmatrix}$$

The four PID design methods, described previously, were successfully applied to the COST simulation benchmark. The dynamic (dry) influent flow conditions has been utilised to assess the each controller’s ability to respond to set-point changes and demonstrated controller’s performance with respect to disturbance rejection.

Table 1 shows a summary of the results obtained for set-point tracking and disturbance rejection using the four control strategies. Fig. 3 shows the closed-loop dynamic performance statistics.

**TABLE 1: DYNAMIC PERFORMANCE**

<table>
<thead>
<tr>
<th></th>
<th>OS(%)</th>
<th>$T_c$ (min)</th>
<th>SSQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>DO3_Dav</td>
<td>6.7</td>
<td>28</td>
<td>4.18e-5</td>
</tr>
<tr>
<td>DO4_Dav</td>
<td>8.3</td>
<td>43.2</td>
<td>3.08e-4</td>
</tr>
<tr>
<td>DO5_Dav</td>
<td>25</td>
<td>57.6</td>
<td>9.8e-4</td>
</tr>
<tr>
<td>DO3_PK</td>
<td>0.7</td>
<td>7.2</td>
<td>3.28e-5</td>
</tr>
<tr>
<td>DO4_PK</td>
<td>2</td>
<td>8.64</td>
<td>2.19e-4</td>
</tr>
<tr>
<td>DO5_PK</td>
<td>15</td>
<td>5.76</td>
<td>9.76e-4</td>
</tr>
<tr>
<td>DO3_PK</td>
<td>0.2</td>
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<td>3.28e-5</td>
</tr>
<tr>
<td>DO4_PK</td>
<td>1.8</td>
<td>8.65</td>
<td>1.65e-4</td>
</tr>
<tr>
<td>DO5_PK</td>
<td>10</td>
<td>8.64</td>
<td>5.9e-4</td>
</tr>
<tr>
<td>DO3_Con</td>
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</tr>
<tr>
<td>DO4_Con</td>
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<td>7.10e-5</td>
</tr>
<tr>
<td>DO5_Con</td>
<td>6</td>
<td>2.80</td>
<td>4.26e-4</td>
</tr>
</tbody>
</table>

The closed-loop response for a set-point change occurs at 8, 10 and 12 days in DO, DO and DO, respectively with the manipulated variable changes in $K_{La3}$, $K_{La5}$, and $K_{La1}$ is shown in Fig.4. Notice how the controller tries to compensate for the set-point changes and disturbances propagated through the system.

Fig. 3. Dry weather performance statistic
This paper proposes a new multivariable PID tuning method. The application of the proposed method and three other multivariable PID methods to a wastewater process is assessed. All of these methods require information only from simple step or frequency tests. Many other test (for rain and constant influent), not reported for space reasons, indicate that the proposed combined produces sensible results. The methods are based on decoupling the system at different frequency points. The paper proposes DRGA to find the best frequency point for decoupling. It was also proposed to fine-tune the controllers using an optimisation procedure. Finally, extensive simulation studies carried out on a nonlinear model demonstrated that the proposed method gives the best performance.

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REFERENCES