Control of an Anaerobic Upflow Fixed Bed Bioreactor

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Abstract—In this paper the feedback linearization control technique is used for controlling an anaerobic upflow fixed bed reactor, which is used for the degradation of wine distillery waste water. The concentration of the volatile fatty acids is used as the output and the dilution rate is used as the control input. Given the closed loop system to be controlled and the specification for the desired behavior, the control law makes the system to behave correctly. The zero dynamics is found to be stable, guaranteeing that the control technique can be used.

1. INTRODUCTION

In the last two decades, stricter environmental laws have arisen in order to decrease the pollution related to the industrial and urban effluents. This situation has lead to an increase in the use of biological waste water treatment processes and the optimization and control of this systems [1]. The objective of any biologic waste water treatment process is to decompose the organic compounds of an organically polluted stream. It implies to decrease the concentration of Chemical Oxygen Demand (COD) below a value that is specified by the environmental laws.

The organic pollution level in a stream is quantified through the Chemical Oxygen Demand (COD). COD determines the quantity of, which is decomposable into CO₂ and H₂O [2]. Anaerobic digestion as a step in the wastewater treatment is very useful since it produces valuable energy (methane) besides removing the organic pollution from the liquid influent. It is well adapted for concentrated wastes such as agricultural and food industry wastewater. Nevertheless, its main drawback is the easy destabilization, giving rise to the disappearance of the methanogenic bacteria [3]. It is recognized that in order to improve the efficiency in the pollution reduction, advanced control systems have to be added. The control objective is to guarantee that the COD concentration in the output flow is less than the environmental constraints, despite the presence of disturbances. This is accomplished, for example, by manipulating the input liquid flow rate [4]. However the control of anaerobic digestion processes is difficult, since some variables as the concentration of COD, VFA, acidogenic and methanogenic bacteria, are not available for on line measurement. Furthermore, there is poor understanding of the process, since the kinetic parameters are generally unknown and time varying. For this reason virtual sensors or state observers are designed [5], [6].

In the anaerobic digestion there are two operating steady states. The first one corresponds to normal operating conditions. The bacteria remain active, and the concentration of COD and VFA is reduced. The second condition is called the washout [7]. The methanogenic bacteria concentration is reduced to zero, giving rise to the accumulation of VFA [2]. Furthermore, there is no reduction of VFA or production of biogas:

\[ X^e_2 = 0, S^e_2 = S^m_2, q_M = 0 \]  \hspace{1cm} (1)

For this reason, the washout is not a desired condition. It can occur due to big values of the dilution rate or the organic concentration in the input flow [2]. For avoiding this phenomenon the maximum value for the specific growth rate \( \mu_{\text{max}} \) must be bounded [7].

In this work, we use the dynamical model of an anaerobic upflow fixed bed reactor described by four ordinary differential equations. The control input is the dilution rate and the controlled variable is the concentration of volatile fatty acids (VFA), which is the output too. The concentration of COD in the input flow is the disturbance. The output must follow the desired reference. The used technique for controlling the system corresponds to a Input-Output linearization, known too as feedback linearization. Hence, it is required that the output follows the reference, while all states of the system keep bounded. The reason for choosing the VFA as the output is that in the washout the VFA is not reduced. So, in this way the washout condition can be avoided.

The main idea of the feedback linearization consists in transforming the original differential equations for obtaining a fully or partially linear model. Then, a linear differential relation between the output and an equivalent input appears as a result of transformation [8]. After, the equivalent input is designed such that the output follows a desired reference, but keeping the no linearized states bounded, [9]. The stability of the other states can be checked using the normal forms. For this reason, the normal form is calculated in explicit way.

The major contribution of this paper is not the development/ implementation of a control law, since it requires the knowledge of the model, including kinetic parameters and state variables. On the contrary, the contribution is the study/analysis of the stability for the feedback linearization control technique, based on the state space model that describes the anaerobic system. Since the designed controller must guarantee the stability of the whole states in a BIBO sense besides regulating...
the output, it is necessary to study the stability of the internal and zero dynamics. The Lyapunov technique is difficult to use, because of the highly nonlinear model. Thus it is not easy to derive an appropriate Lyapunov function.

This work is organized as follows. In Section II, the system and the corresponding dynamical model are presented. Section III deals with the mathematical procedure for obtaining the partially linearized model. The controller is calculated in this part too. In Section IV, the transformations to the normal form are shown. Section V shows the obtained results after applying the designed controller to the dynamical model. Very good results are obtained despite the presence of a disturbance. In the final part (Section VI) some conclusions and future work are presented.

II. PHYSICAL SYSTEM

The anaerobic digestion is a microbiologic process carried out in absence of oxygen. The organic matter is decomposed and it generates: a gas (called biogas which is highly energetic), a residual sludge, and a wastewater with less pollution [10]. A general sketch of the anaerobic upflow fixed bed reactor is shown in fig. 1.

The chosen dynamical model is presented in [1], [11].

![Fig. 1. Sketch for the anaerobic upflow fixed bed reactor](Image)

The state of the system is given by \( X_1(t), X_2(t), S_1(t), S_2(t) \), which are the state variables of the system and represent concentrations: the acidogenic biomass concentration, the methanogenic biomass concentration, the COD concentration and the VFA concentration, respectively. Then the dynamical model is:

\[
\begin{align*}
\dot{X}_1 &= (\mu_1 - \alpha D) X_1 \\
\dot{X}_2 &= (\mu_2 - \alpha D) X_2 \\
\dot{S}_1 &= (S_1^{in} - S_1) D - k_1 \mu_1 X_1 \\
\dot{S}_2 &= (S_2^{in} - S_2) D + k_2 \mu_1 X_1 - k_3 \mu_2 X_2 \\
\end{align*}
\]

(2)

where:

\[
\begin{align*}
\mu_1 &= \mu_{max}1 \frac{S_1}{K_{S1} + S_1} \\
\mu_2 &= \mu_0 \frac{K_{S2} + S_2}{K_{S2} + S_2 + \left( \frac{S_2^{in}}{K_{I2}} \right)^2} \\
\end{align*}
\]

(3)

The used nomenclature and the meaning of the letters are presented in Tab. I.

III. INPUT OUTPUT LINEARIZATION

Usually the COD concentration \( S_1 \) is the controlled variable in order to keep the pollution low. However, in this paper, the controlled variable is the VFA concentration, corresponding to variable \( S_2 \). It is assumed that this variable is available for measuring. By controlling the VFA it is possible to guarantee a desired methane production. The control input is the dilution rate \( D \), which is the ratio between the input flow rate divided by the volume of the liquid inside the reactor. Then, the final control element is a valve that handles the input flow rate. The COD concentration in the input flow \( S_1^{in} \) is the disturbance. The used control technique corresponds to feedback linearization, for obtaining a fully or partially linear model. Similar strategies can be found in [10], [12].

Consider the nonlinear system with a single input and a single output, described by the following state space

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning and value</th>
</tr>
</thead>
<tbody>
<tr>
<td>VFA</td>
<td>Volatile fatty acids.</td>
</tr>
<tr>
<td>COD</td>
<td>Chemical oxygen demand.</td>
</tr>
<tr>
<td>( D )</td>
<td>Dilution rate [day(^{-1})]</td>
</tr>
<tr>
<td>( X_1 )</td>
<td>Acidogenic biomass concentration [g/L].</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>Methanogenic biomass concentration [g/L].</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>COD concentration [g/L].</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>VFA concentration [mmol/L].</td>
</tr>
<tr>
<td>( S_1^{in} )</td>
<td>COD concentration in the input flow ( S_1^{in} = 5.8 ) [g/L].</td>
</tr>
<tr>
<td>( S_2^{in} )</td>
<td>VFA concentration in the input flow ( S_2^{in} = 52 ) [mmol/L].</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>Acidogenic biomass growth rate [day(^{-1})].</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>Methanogenic biomass growth rate [day(^{-1})].</td>
</tr>
<tr>
<td>( \mu_{max1} )</td>
<td>Maximum acidogenic biomass growth rate ( \mu_{max1} = 1.2 ) [day(^{-1})].</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>Parameter associated with the maximum methanogenic biomass growth rate ( \mu_0 = 0.74 ) [day(^{-1})].</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>Yield coefficient for the COD degradation ( k_1 = 10.53 ) [g COD/g ( X_1 )].</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>Yield coefficient for the VFA production ( k_2 = 28.6 ) [mmol VFA/g ( X_1 )].</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>Yield coefficient for the VFA consumption ( k_3 = 1074 ) [mmol VFA/g ( X_2 )].</td>
</tr>
<tr>
<td>( K_{S1} )</td>
<td>Saturation parameter associated with ( S_1 ) ( K_{S1} = 7.1 ) [gCOD/l].</td>
</tr>
<tr>
<td>( K_{S2} )</td>
<td>Saturation parameter associated with ( S_2 ) ( K_{S2} = 9.28 ) [mmolVFA/l].</td>
</tr>
<tr>
<td>( K_I )</td>
<td>Inhibition constant associated with ( S_2 ) ( K_I = 16 ) [mmolVFA/l].</td>
</tr>
</tbody>
</table>
representation:
\[ \dot{x} = f(x) + g(x)\ u \]
\[ y = h(x) \]  
(4)

where \( u \) is the input and \( y \) is the output of the system. Taking into account this representation, we have:
\[ f(x) = \begin{bmatrix} \mu_1 X_1 \\ -k_1 \mu_1 X_1 \\ \mu_2 X_2 \\ k_2 p_1 X_1 - k_3 p_2 X_2 \end{bmatrix}, \quad g(x) = \begin{bmatrix} -\alpha X_1 \\ S_1^{in} - S_1 \\ -\alpha X_2 \\ S_2^{in} - S_2 \end{bmatrix} \]  
(5)

where \( h(x) = S_2 \) and \( D = u \).

Using feedback linearization, we obtain a linear differential equation between the output \( y \) \((S_2)\) and a new input \( \nu \).

A useful concept for applying feedback linearization is the relative degree \((r)\). The relative degree is the number of times that the output is derived until the input \( u \) appears in an explicit way. For applying input output linearization it is necessary to derive the output \( r \) times. In this way, the input \( u \) appears. Then, the input \( u \) is designed for cancelling the nonlinearities.

If \( L_\nu h(x) \neq 0 \) for some neighborhood of \( x_0 \) in the space \( \Omega_x \), the relative degree is 1. In this case, the following input transformation leads to a linear relation between the output and the new input \( \nu \):

\[ \dot{\nu} = \nu \]
\[ u = \frac{1}{S_2}(\nu - L_\psi h + \psi) \]  
(6)

\( L_\nu h(x) \) is known as the Lie derivative of \( h \) with respect to \( f \), and it is evaluated from: \( \nabla h \cdot g \)

In our system, \( L_\nu h \) is:

\[ L_\nu h = \nabla h \cdot g = \begin{bmatrix} \frac{\partial h}{\partial X_1} & \frac{\partial h}{\partial S_1} & \frac{\partial h}{\partial X_2} & \frac{\partial h}{\partial S_2} \end{bmatrix} \begin{bmatrix} -\alpha X_1 \\ S_1^{in} - S_1 \\ -\alpha X_2 \\ S_2^{in} - S_2 \end{bmatrix} \]  
(7)

\[ L_\nu h = S_2^{in} - S_2 \neq 0 \]

Therefore, the relative degree of the system is 1 and the input output relation is as in equation 6. The generalization of this technique for other relative degrees can be found in [13], [14]. The control input is calculated according to equation 7 [15] as:

\[ D = \frac{1}{S_2^{in} - S_2}(-k_2 \mu_1 X_1 + k_3 p_2 X_2 + \nu) \]  
(8)

where \( \nu \) is the new input, which is chosen such that the linear system is stable. For this case, due to the relative degree is 1 and by avoiding the use of other controller, it is sufficient to select \( \nu \) as:

\[ \nu = K_p(S_2 - S_2^{ref}) + \xi^{ref} = K_p e + \xi^{ref} \]  
(9)

where \( S_2^{ref} \) is the reference or desired value for the output, \( e \) is the difference between the output and the reference and \( K_p \) is a constant. Replacing equations 9 and 10 in the fourth equation of our model (equation 2), we obtain the following linear input output relation:

\[ \frac{de}{dt} = K_p e \]  
(10)

Therefore, the dynamics of the tracking error is stable for \( K_p < 0 \) and the output tracks the reference.

However for successful application of the controller it is necessary that not only the output follows a desired trajectory, but the whole states remain bounded. In the input output linearization, when \( r < n \), only a part of the nonlinear system is linearized, but the other part is uncontrollable. This part is called the internal dynamics. Then, it is necessary to study the stability of the internal dynamics, in order to guarantee that the states remain bounded during the tracking. Since determining the stability of the internal dynamics is very difficult, a simpler way for doing this is the so called zero dynamics.

The zero dynamics is an intrinsic feature of a nonlinear system, which does not depend on the control law or the desired trajectories, but only depends on the internal states.

For studying the zero dynamics and determining the stability of the internal dynamics we transform the system into a normal form.

IV. NORMAL FORMS

The objective of the normal form is to take the system into a representation where the internal dynamics may be evaluated in a simpler way. Let be

\[ v = [v_1 \ v_2 \ldots v_r]^T = [y \ \dot{y} \ \ldots \ \dot{y}^{r-1}]^T \]  
(11)

In a neighborhood \( \Omega \) of a point \( x_0 \), the normal form of the system is:

\[ \dot{v} = \begin{bmatrix} v_2 \\ \vdots \\ v_r \\ f_1(v, \psi) + f_2(v, \psi) D \end{bmatrix} \]  
(12)

\[ \dot{\psi} = w(v, \psi) \]  
(13)

where \( v \) and \( \psi \) are the new states in \( \Omega \) and \( y \) is the output. Equation 13 is known as the normal form. For obtaining a real state transformation, the gradients \( \nabla \mu_i \) and \( \nabla \psi_j \) must be linearly independent. The vector fields \( \psi_j \) must satisfy the next relation:

\[ \nabla \psi_j \cdot g = 0 \text{ for all } 1 \leq j \leq n - r \]  
(14)

For the studied system, we have \( r = 1 \) and \( n = 4 \), so there will be three components of the vector fields, noted as \( \psi_j \). From equation 12, \( v = y = S_2 \), \( \dot{v} = \dot{S}_2 \). Now, the
other components of the vector field $\psi_j$ can be calculated from:

$$
\begin{bmatrix}
\frac{\partial \psi_1}{\partial x_1} & \frac{\partial \psi_1}{\partial x_2} & \frac{\partial \psi_1}{\partial S_1} & \frac{\partial \psi_1}{\partial S_2}
\end{bmatrix} \begin{bmatrix}
\alpha X_1 - \alpha X_2
\end{bmatrix} = 0 \quad (15)
$$

Solving this equation leads to:

$$
\begin{align*}
\dot{\psi}_1 &= \frac{1}{\alpha} \ln X_2 + \ln (S_1^n - S_2) \\
\dot{\psi}_2 &= \frac{1}{\alpha} \ln X_1 + \ln (S_1^n - S_2) \\
\dot{\psi}_3 &= \frac{1}{\alpha} \ln X_1 + \ln (S_1^n - S_1) \quad (16)
\end{align*}
$$

The gradients $\nabla \mu_i$ and $\nabla \psi_j$ must be independent. The linear independence is proved through the determinant of the Jacobian. The determinant must be non-zero. That is:

$$
\begin{bmatrix}
\frac{\partial \psi_1}{\partial x_1} & \frac{\partial \psi_1}{\partial x_2} & \frac{\partial \psi_1}{\partial S_1} & \frac{\partial \psi_1}{\partial S_2} \\
\frac{\partial \psi_2}{\partial x_1} & \frac{\partial \psi_2}{\partial x_2} & \frac{\partial \psi_2}{\partial S_1} & \frac{\partial \psi_2}{\partial S_2} \\
\frac{\partial \psi_3}{\partial x_1} & \frac{\partial \psi_3}{\partial x_2} & \frac{\partial \psi_3}{\partial S_1} & \frac{\partial \psi_3}{\partial S_2}
\end{bmatrix}
$$

where $z = [\psi_1 \psi_2 \psi_3]^T$, then

$$
\left| \frac{\partial z}{\partial \mu} \right| = \frac{1}{\alpha^2 X_1 X_2 (S_1^n - S_1)} \quad (17)
$$

It is easy to see that this condition is always fulfilled, and then the inverse transformation is given by:

$$
\begin{align*}
X_1 &= \left( \frac{\psi_1 + \psi_2}{S_2^n - \psi_2} \right)^{-\alpha} \\
X_2 &= \left( \frac{\psi_1}{S_2^n - \psi_2} \right)^{-\alpha} \\
S_1 &= S_1^n - (S_2^n - \psi_2) e^{\psi_3 - \psi_2 - \psi_1} \\
S_2 &= \psi_2
\end{align*} \quad (19)
$$

According to this new coordinates, and replacing equations 9 and 10, we obtain the following equations:

$$
\begin{align*}
\dot{v} &= K_p (v_2 - v_2^{ref}) + \dot{v}_2^{ref} \\
\dot{\psi}_1 &= \frac{1}{S_2^n - \psi_2} \left( k_3 \mu_1 \left( \frac{\psi_1 + \psi_2}{S_2^n - \psi_2} \right)^{-\alpha} - k_2 \mu_1 \left( \frac{\psi_1 + \psi_2}{S_2^n - \psi_2} \right)^{-\alpha} \right) - \frac{\dot{\mu}_2}{\alpha} \\
\dot{\psi}_2 &= \frac{\dot{\psi}_2}{\alpha} + \frac{\Dot{\mu}_1}{\alpha} \left( \frac{\psi_1 + \psi_2}{S_2^n - \psi_2} \right)^{-\alpha} \\
\dot{\psi}_3 &= -\frac{\dot{\mu}_1}{\alpha} + \frac{k_3 \mu_1}{(S_2^n - \psi_2)^{\alpha}} \left( \frac{\psi_1 + \psi_2}{S_2^n - \psi_2} \right)^{-\alpha} \\
\end{align*} \quad (20)
$$

where

$$
\begin{align*}
\dot{\mu}_1 &= \mu_{max 1} \frac{S_1^{(n)} - (S_2^{(n)} - \psi_2 - \psi_1)}{S_1^{(n)} + S_1^{(n)} - (S_2^{(n)} - \psi_2 - \psi_1)} \\
\dot{\mu}_2 &= \mu_2 + K_2 v_2 (\psi_2^{ref})^7 \\
\end{align*} \quad (21)
$$

The first equation corresponds to the input-output relation and the other ones correspond to the internal dynamics. When the input is such that the output becomes zero, the internal dynamics is called zero dynamics. If the zero dynamics is stable, the internal dynamics is stable too. The zero dynamics is obtained by letting $v = 0$:

$$
\begin{align*}
\dot{\psi}_1 &= -k_3 \mu_1 \left( \frac{\psi_1 + \psi_2}{S_2^n - \psi_2} \right)^{-\alpha} \\
\dot{\psi}_2 &= -\frac{\dot{\mu}_1}{\alpha} \\
\dot{\psi}_3 &= -\frac{\dot{\mu}_1}{\alpha} + k_3 \mu_1 \left( \frac{\psi_1 + \psi_2}{S_2^n - \psi_2} \right)^{-\alpha} \\
\end{align*} \quad (22)
$$

where

$$
\mu_1 = \mu_{max 1} \frac{S_1^{(n)} - (S_2^{(n)} - \psi_2 - \psi_1)}{S_1^{(n)} + S_1^{(n)} - (S_2^{(n)} - \psi_2 - \psi_1)} \quad (23)
$$

Proving the stability of this set of equations is a very hard work. However, it is possible to determine the equilibrium points and apply a stability theorem. For example, next section deals with the stability of one of the equilibrium points.

V. RESULTS

The behavior of the zero dynamics, corresponding to equation 22 is shown in figs. 2, 3 and 4. These figures show the stability of the internal dynamics. The variables converge to $(\psi_1, \psi_2, \psi_3) = (-8.525, -4.3, 2.03)$

As we said previously the zero dynamics is stable, that means that the internal dynamics is stable too. Therefore, the designed controller can be applied.

Simulations of the system described by equation 2 and controlled with equations 9 and 10 are shown in figs. 5, 6, 7, and 8. Initially the system is in open loop with a constant value of the input $D = 0.36$ day$^{-1}$. At time $t = 60$ days the control technique is turned on and the system works in closed loop. The set point is $S_2^{ref} = 2.4$ [mmol/l]. At time 120 days, a disturbance is introduced and it is represented in a variation of the parameter $S_1^{(n)}$. This parameter changes from 5.8 to 6.4 [g/ℓ]. The control parameter $K_p$ has a value of $K_p = 2$. As can be seen, the output follows the reference when the closed loop is used, and the disturbance does not affect the output. Fig. 5
VI. CONCLUSIONS

The conditions corresponding to feedback linearization were obtained and applied to anaerobic upflow fixed bed reactor. Several simulations showed the advantages of using Feedback linearization. First, a linearizing control law was obtained, and a linear relation between the the input and the output of the system. Then, the normal form was found through a nonlinear transformation, obtaining the internal dynamics. Finally, the internal dynamics were studied through simulation, guaranteeing the system stability. This showed a nice possibility of applying the designed controller. Finally, the controller was implemented on the system, obtaining good results.

REFERENCES


