Compensation of jammed control surface of large transport aircraft by control reconfiguration

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Abstract—This paper addresses the control allocation to several aircraft flight controls to produce required body axis angular accelerations. Two methods of control law design are used to produce the virtual control effort signals, which are then distributed by solving sequential least squares problem using active set method to the flight control surfaces to generate this effort. Two cases are described: in the first case the control law and allocation for the healthy aircraft is implemented, and in the second case, jamming of one control surface is introduced at time zero. In this case it was shown that how controller and allocation compensate for this failure without changing the control law. To implement this system it was assumed that there is a good fault identification system onboard. Normally aircraft are over-actuated and in the case of a control failure this over actuation is more pronounced due to coupling of aircraft dynamics. The problem of control allocation presented here is a convex problem so there is always a unique solution.

I. INTRODUCTION

Modern jet aircraft have many actuators required for the flight path control (e.g. two or more engines, elevators, rudders, flaps). In essence aircraft are "over-actuated" as they possess control redundancy and the pilot commanded flight vector can be realized with more than one (often many) different combinations of settings of the actuators. With advanced control schemes this redundancy in the control of actuators can be taken advantage of to enhance aircraft safety in the event of an aircraft malfunction or damage. The research objective is to utilize the multiple redundancies in the control systems in the event of a system failure or other aircraft malfunction to control the aircraft by automatically switching control laws and control allocation techniques. This technique, which is based on online optimization, has recently been explored for use in military air vehicles; however, little research has been undertaken for civil aircraft applications.

Gao and Antsaklis [1] has proposed the approach of control law reconfiguration using Pseudo Inverse Method (PIM), which was successfully accepted in flight simulation by Caglayan et al [2]. The main idea is to modify feedback gain so that the reconfigured system approximate to the nominal system in some sense. They proposed the modified PIM with respect to stability constraint, this method loses optimal sense in dealing with the general multivariable systems. This limitation was overcome in robust control mixer module method by Yang and Blanke [3]. The mentioned approaches related to control mixer method deals with configuring flight control law. In control allocation (CA) Härkegård [4], control design is separated in following two steps

- Control law specifying which total control effort to be produced (net torque, force, etc.).
- Design a control allocator that maps the total demand onto individual actuator settings (commanded aerosurfaces deflections, thrust, forces, etc.).

Design a Geometric constrained control allocation was proposed under the assumption that the actuators are linear in their effect throughout their ranges of motion and independent from one another in their effects Durham [5]. This was extended to three moment problem by Durham [6]. Linear and quadratic programming approaches for control allocation are given in Dale [7]. Evaluation of optimization methods for control allocation by Bodson [8] discusses a variety of issues that affect the implementation of various algorithms in a flight control systems. Comparison between robust servomechanism and CA was done by Burken et al. [9] with later working with fault detection system. The concept of static CA was modified to dynamic CA by Härkegård [10]. Karen and Krishnakumar [11] proposed control reallocation strategies with daisy chain CA, optimal CA using linear programming and table look up with blending. Comparison of optimal versus CA was shown in Härkegård [12]. Doman et al. [13][14] work on dynamic control allocation with non-negligible actuator dynamics. Historically control allocation has been performed by assuming that a linear relationship exists between the control induced moments and the control effectors displacements. However a non-linear relationship leads to non-linear CA, as proposed by Doman et al. [15][16].

II. MODELING OF B747 100/200

A dynamic rigid body model of the Boeing 747 is considered in this paper. The Simulink model for this aircraft is FTLAB747 [17] and [18]. Originally, in the flight control system of FTLAB747 there is no actuator redundancy utilization and similar control surfaces are considered to be the same (e.g. four elevators are considered one surface). In the design of control allocation scheme all actuators redundancy should be exploited. The sign convention used for control surface position is leading edge down is treated as positive. Input to the aerodynamic model is $u_{aero}$, which is a control vector of eleven control signals to the block. Input to the propulsion system is $u_{prop}$.

$\delta_{cor}$, $\delta_{sol}$, $\delta_{air}$ and $\delta_{cil}$ are right/left outboard and right/left inboard ailerons respectively. $\delta_{cor}$, $\delta_{sol}$, $\delta_{air}$ and $\delta_{cil}$ are right/left outboard and right/left inboard elevators.
respectively. $\delta_{ih}$ is stabilizer, $\delta_{ur}$ and $\delta_{dr}$ upper and downward rudders respectively.

$T_{n1}, T_{n2}, T_{n3}$ and $T_{n4}$ are thrust of engines 1, 2, 3, 4 respectively.

$u_{\text{aero}} = \begin{bmatrix} \delta_{aor} & \delta_{aol} & \delta_{air} & \delta_{ail} & \delta_{eir} & \delta_{eil} & \delta_{ih} & \delta_{ur} & \delta_{dr} \end{bmatrix}^T$

$u_{\text{prop}} = \begin{bmatrix} T_{n1} & T_{n2} & T_{n3} & T_{n4} \end{bmatrix}^T$

III. TRIMMING AND LINEARIZATION

This model of aircraft is trimmed at straight and level flight with flight condition of 241 m/s true airspeed and 7000 m height with the flight path angle $\gamma$ set to zero. The trimmed flight control (radians) and thrust control (newtons) vectors are

$u_{\text{aero, trim}} = \begin{bmatrix} 0.003 & 0.003 & -0.005 & -0.005 & 0 & 0 & 0 & 0.007 & 0 & 0 \end{bmatrix}^T \text{rad}$

$u_{\text{prop, trim}} = \begin{bmatrix} 43315.805 & 43315.805 & 43315.805 & 43315.805 \end{bmatrix}^T \text{N}$

The aircraft is linearized around this searched equilibrium point by introducing the deviation $\Delta x = x - x_{\text{trim}}$ and $\Delta u = u - u_{\text{aero, trim}}$

$\Delta \dot{x} = A\Delta x + B_u\Delta u$

$\Delta y = C\Delta x + D\Delta u$ (1)

where $x \in \mathbb{R}^n$ is the system state $u \in \mathbb{R}^m$ is the control input to the system, and $y \in \mathbb{R}^p$ is the output of the system to controlled.

The state vector is

$x = \begin{bmatrix} \alpha & \beta & p & q & r \end{bmatrix}^T$

where

$p,$ is the roll rate about body x-axis (rad/s)
$q,$ is the pitch rate about body y-axis (rad/s)
$r,$ is the yaw rate about body z-axis (rad/sec)
$\alpha,$ is the angle of attack (rad)
$\beta,$ is the side slip angle (rad)
$\phi,$ is the roll angle (rad)

IV. CONTROL ALLOCATION

Control allocation is useful for the control of over-actuated systems, and deals with distributing the total control demand among the individual actuators. Using control allocation, the actuator selection task is separated from the regulation task in the control design[4]. To introduce the ideas behind control allocation, consider the following system

$x = u_1 + u_2$

Where $x$ is a scalar state variable, and $u_1$ and $u_2$ are control inputs. $x$ can be thought of as the velocity of a unit mass object affected by a net force $v = u_1 + u_2$ produced by two actuators.

Assume that to accelerate the object, the net force $v = u_1 + u_2$ produced by two actuators. Assume that to accelerate the object, the net force $v = 1$ is to be produced. There are several ways to achieve this. We can choose to utilize only the first actuator and select $u_1 = 1$ and $u_2 = 0$ or to gang the actuators and use $u_1 = u_2 = 0.5$. We could even select $u_1 = -12$ and $u_2 = 11$ although this might not be very practical. Which combination to pick is essentially the control allocation problem. (Today, control allocation is a research topic in aerospace control and marine vessel control).

The nominal control allocation layout for healthy aircraft is shown in Figure (1).

In Figure 1 $v$ is virtual control signal to the control allocation part.

The control law is designed separately from the allocation scheme. Control law are designed using linear quadratic regulator (LQR) design. Two control laws designs are proposed.

V. CONTROL LAW DESIGN

The control law for the linear model given in Eq. (1) is designed with virtual control signals of angular acceleration in roll, pitch and yaw.

The input matrix $B_u$ is factored in to $B_u = B_{\text{vir}}B$

where the rank$(B_u) = k \leq m$ and $B_u$ is $n \times m$, $B_{\text{vir}}$ is $n \times k$, and $B$ is $k \times m$.

The system in Eq.(1) is now given by

$\dot{x} = Ax + B_{\text{vir}}v$ (2)

$v = Bu$ (3)

$y = Cx$ (4)

Note that $\Delta$ is removed for simplicity of notation.

The virtual control input is determined by solving

$$\min_v \int_0^\infty ((x - x^*)Q(x - x^*) + (v - v^*)R(v - v^*))dt$$ (5)

where $x^*, v^*$ solve following problems, resulting in a good trajectory tracking performance and the dynamics of aircraft to be in steady state.

$Ax + B_{\text{vir}}v = 0$

$Cx = r$

and $Q$ is positive semidefinite and $R$ positive definite matrices. The above Eq. (5) is LQR problem, with $v = \begin{bmatrix} p & q & r \end{bmatrix}^T$ and $y = \begin{bmatrix} \alpha & \beta & p \end{bmatrix}^T$

The virtual control input is given by

$v(t) = K_r r + K x(t)$

$K_r = H_0^{-1}$

$K = R^{-1}B_{\text{vir}}^T P$

Where

$H_0 = C(B_{\text{vir}}K - A)^{-1}B_{\text{vir}}$
and $P$ is the unique positive semi definite and symmetric solution to the Riccati equation

$$A^TP + PA + Q - PB_{vir}R^{-1}B_{vir}^TP = 0$$

Proof of the above is given in reference [4]. This control law design is the baseline control law and the optimal virtual control inputs are distributed optimally and feasibly among different control surfaces using control allocation.

VI. CONTROL LAW DESIGN 2

The second design is based on robust servomechanism design, which is generalization of proportional-plus-integral (PI) design. A PI controller is designed to stabilize the aircraft (stabilization) [9]. This law is also treated as baseline control law, and in the event of failure the redundant degrees of freedom are utilized to cancel the effect of the jammed surface using control allocation.

The controller dynamics are set to be

$$\dot{x}_c = A_c x_c + B_c (r - y) \quad (6)$$

where $x_c \in \mathbb{R}^p$ is the controller states, $A_c \in \mathbb{R}^{p \times p}$ and $B_c \in \mathbb{R}^{p \times p}$. Consider the open loop system including the plant Eqs. (2-4) and Eq. (6) with $r = 0$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A & 0 \\ -B_c C & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} B_{vir} \\ -B_c D \end{bmatrix} v \quad (7)$$

with $v = [p \ q \ r]^T$ and $y = [\alpha \ \beta \ p]^T$. The controllability of the augmented system Eq. (7) is checked by

$$\text{rank}(C_0) = l$$

where

$$C_0 = [B_c \ A_c B_c \ A_c^2 B \ \cdots \ A_c^{l-1} B_c]$$

$n + p$ and $l = n + p$.

The augmented system Eq. (7) is controllable. Hence there exist control laws

$$v = k_x + k_c x_c \quad (8)$$

such that the closed loop system is stable.

The control law can be conveniently found by applying LQR approach to Eq. (7). In this special case $r$ is a constant command therefore, $A_c = [0_{3 \times 3}]$ and $B_c = I_{3 \times 3}$, according to their definitions. From controller dynamics Eq. 6 it can be seen that $x_c = \int (r - y) dt = \int c dt$. Thus control law Eq. (8) is simply a PI control law of multi input and multi output (MIMO) system.

The structural design limitation in terms of load factor for the B747 is considerably smaller than a high maneuverable fighter aircraft. So it is better to control the position rather than the rotation rates (i.e. roll, pitch, and yaw). In this way the aircraft will easily remain inside the design limits. To add this limitation to the control laws (Design 1 and Design 2) an additional outer position controller is cascaded to exiting design to control the roll angle as shown in Figure 2.

And now the reference vector is $r = [\alpha_{ref} \ \beta_{ref} \ \phi_{ref}]$, the output vector would be $y = [\alpha \ \beta \ \phi \ p \ q \ r]^T$ and additional state $\phi$ is added to the state vector $x = [\alpha \ \beta \ \phi \ p \ q \ r]^T$.

Now the control allocation is designed using sequential least squares (SLS) to distribute the virtual control effort $v = [p \ q \ r]^T$ among the control surfaces optimally and feasibly.

VII. CONTROL ALLOCATION PROBLEM FORMULATION

In control allocation the reference baseline control law is first designed for the healthy aircraft as described in the previous section. In this section control allocation algorithm will be shown. Control allocation problem is formulated as

1) Problem formulation: The pre failure state equation is given by Eqs. (2-3). Another output variable $z$ is introduced.

$$z = C_z x$$

Post failure output dynamics

$$\dot{z} = C_z A x + C_z B_{vir} v + C_z b_w w \quad (10)$$

We have to seek $u_r$ so that

$$v \approx B_r u_r \quad (11)$$

to compensate for constant disturbance due to the jammed control surface.

Where $u_r$ are the remaining control surfaces, $w$ is constant position of run away and jamming of control surface and $b_w$ is the corresponding column of the jammed actuator in control effectiveness matrix $B_w$.

To simplify the problem the actuator dynamics are neglected. Control allocation is under the assumption that there is a good fault identification system available. The control allocation in failure case is given by Figure 3.

A. SLS using active set methods

A sequential treatment of Least Squares problems may be preferable for several reasons:

- It divides a large computing burden into smaller parts, to reduce the requirements on both processing capability and storage, and
- It is the key to real-time applications.

Active set methods are used in many of today’s commercial solvers for constrained quadratic programming and can be shown to find the optimal solution in a finite number of iterations. In this method the inequality constraints are either disregarded or treated as equality constraints. The algorithm comprises two phases [19].

1) Phase 1:

a. At the start $u_0 = \frac{u_{\text{max}} + u_{\text{min}}}{2}$ and $W = []$, otherwise it is the working set of active equality constraints from previous sampling.

b. Here the post failure Eq. (??) is used in allocation problem, solve $l_2$-optimal control allocation problem

$$u_{\Omega} = \arg \min_{u_r} \| W_{vir} (B_r u_r - v) \|_2 \quad (12)$$

subject to $B_r u_r = v \quad (13)$
The problem Eqs. (12,13,14) can be written as

\[ \min_{u_r} \| A u_r - b \|_2 \]

where

\[ A = W_{v\alpha r} B_r \quad b = W_{v\beta r} v \]

\[ \min_{\tilde{p}} \| A(u_r^i + \tilde{p}) - b \|_2 \]  

\[ B_r \tilde{p} = 0 \]  

where \( \tilde{p} \) is the optimal perturbation such that moving along \( \tilde{p} \) from \( u_r^i \), \( B_r u_r^i \) does not change because

\[ B_r(u_r^i + \tilde{p}) = B_r u_r^i = v \]

The iterative algorithm is as follows:

for \( k = 0,1,2,3,... \)

solve (16,17) to find \( p_i^k \) where \( i \in W_k \cap I \)

if \( p_i^k = 0 \)

compute Lagrange multiplier \( \lambda_i \) from

\[ A^T (Au - b) = \begin{bmatrix} B_r^T & C_0^T \end{bmatrix} \begin{bmatrix} \mu \\ \lambda \end{bmatrix} \]

\( C_0 \) is active constraints in \( W_k \), \( \mu \) is associated with Eq. (13) and \( \lambda \) with the active constraints in Eq. (14).

if \( \lambda_i \geq 0 \) for all \( i \in W_k \cap I \)

STOP with solution \( u_r = u_r^k + 1 = u_r^k + p_i^k \);

else

Set \( j = \arg \min \lambda_j \);

\( u_{r,1}^{k+1} = u_r^k + p_i^k; W_{k+1} = W_k \)

\( (\text{Dropping the constraint } j \text{ from } W_k) \)

else \((p_i^k \neq 0)\)

compute \( \alpha_k \)

\[ u_{r,1}^{k+1} = u_r^k + \alpha_k p_i^k \]

If there are blocking constraints obtain \( W_{k+1} \) by adding one of the blocking constraints to \( W_{k+1} \)

else \( W_{k+1} = W_k \); 

end for

\( c. \) If \( B_r u_{\Omega} = v \), move to phase 2 else stop with \( u = u_{\Omega} \).

2) Phase 2:

a. Let initial \( u^0 = u_{\Omega} \) and \( W \) is the working set from phase 1

b. Solve

\[ u_r = \arg \min_{u_r} \| W_u (u_r - u_d) \| \]

\[ B_r u_r = v \]

\[ u_{\min} \leq u_r \leq u_{\max} \]

using the algorithm given above.

The weighting matrices \( W_{\alpha} \) and \( W_{\beta} \) are assumed to be non-singular. \( W_u \), being non-singular ensures that the posed optimization problems have unique optimal solution. In phase 2 of SLS the desired control surface input \( u_d \) are treated as zero to achieve minimum drag.

VIII. SIMULATION RESULTS

A. Retrim and stability

Before proceeding with the control allocation for damage adaptation, it is important to determine whether the aircraft can still be retrimmed with a particular aerosurface jammed at a given position. This approach is taken from [9]. Rewrite the post failure aircraft model as

\[ \dot{x} = Ax + B_r u_r + b_0 \delta \]  

where \( \delta \) is the jammed surface position (as before), \( B_r \) is the post failure \( B_u \) matrix, \( u_r \) is the remaining control surfaces, and \( b_0 \) is the control effectiveness vector corresponding to the jammed surface.

Fig. 2. Control design with roll angle to be controlled.

Fig. 3. Control allocation under the assumption of fault diagnostic system onboard.
Let \( y_d \) represent the three body angular (roll, yaw, and pitch) rates of the vehicle in body frame. Suppose that \( y_d = C_d x \), then
\[
\dot{x} = C_d x + C_d B_r u_r + C_d b_\delta \delta \quad (19)
\]
A necessary condition for retrimming the vehicle with the jammed surface is that the right hand side of the preceding equation can still be made to vanish at \( x = 0 \) with \( u_r \) in its allowable range. The range of jammed position \( \delta \) for which retrim is possible. Following linear programming (LP) problem is solved.
\[
\min \delta \quad \text{or} \quad \max \delta
\]
subject to
\[
C_d B_r u_r + C_d b_\delta \delta = 0 \quad (21)
\]
\[
u_r \min \leq u_r \leq u_r \max \quad (22)
\]
\[
\delta \min \leq \delta \leq \delta \max \quad (23)
\]

The solution of the LP problem Eqs.(20 to 23) gives the minimum (most negative) or maximum jammed incremental position of \( \delta \) that can be balanced at the trim condition by the remaining aerosurfaces within the saturation limits. This range found serves as a reasonable estimation within which the reconfigurable control can still possibly stabilize the system. In the present study, which concerned an aileron jamming at full range (-20 to 20) degrees, it is required that the system can still be retrimmable and stabilizable.

Closed loop stability is assured by constructing a control law, in terms of the virtual control signal, which stabilizes the system (sufficient condition). The control allocator merely distributes the total control demand among the available effectors and (in principle) does not affect the closed loop behavior. However, if the control demand from the control law cannot be fulfilled, closed loop stability cannot be assured but the system does not necessarily become unstable. In this case control demand is fulfilled by the control laws. And the nominal control law does not push the vehicle too hard for performance. Hence, the control laws were designed in terms of virtual control signals.

B. Control allocation with jamming of control surface

Damaged is introduced by jamming the left inboard aileron at 20 degrees downward at time zero, which acts like the flap as can be seen by the loss angle of attack. In high speed flight the outboard ailerons are neutral [18] (due to the inherent torsional elasticity of wings). This limitation is taken into account by penalizing the outboard ailerons in the control allocation of the rigid body dynamics of aircraft (this can be seen in Figure 5).

The inboard right aileron will move to maintain symmetry, but the rest of rolling moment compensation is coming from the rudders because of strong coupling of lateral and directional dynamics. As longitudinal dynamics is less coupled to lateral/directional dynamics, so the required angle of attack is achieved by symmetrical deflection of elevators.

The solution is a unique one as the problem is convex. So there are no local minimums and thus there is no chattering to other local minimums and the only local minimum is the global minimum. The spikes in Design 1 shown in Figures 5 and 6 are due to the fact there is no actuator dynamics only one to one mapping from control allocation block to control surfaces is presented here. Design 2 is better due to slow dynamics and the solution does not show spikes.

As can be seen from Figure 4 the Design 1 control law has limitation in terms of a steady state error of directional/lateral states (\( \beta \) and \( \phi \)), but the longitudinal dynamics is working well as there is less coupling between lateral and longitudinal dynamics in this stage. This problem is circumvented by using Design 2 strategy which results in no steady state error due to the integral action of the controller. But Design 2 resulted in slower response of angle of attack.

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IX. CONCLUSION

This research work mainly focused on flight safety. It can be seen from the results that the control allocation
has found the optimal and feasible solution (flight control settings) by solving SLS problem using the active set method. The control law Design 2 and the associated settings) by solving SLS problem using the active set has found the optimal and feasible solution (flight control design methods: Robust servomechanism and control allocation, Journal of Guidance, Control, and Dynamics, vol. 16, no. 4, p. 717725, 1993.


In future work, the EL 1862 B747-200A freighter crash near Amsterdam Schiphol Airport (1992) will be used as a case study. In this crash the aircraft encountered multiple wing engine separation on the right wing [21]. For this study control allocation will be designed to exploit the remaining control surfaces and thrust vector, after fixing the bounds of controllability.