An Adaptive Law for Slope Identification, Position Tracking and Force Regulation for a Robot in Compliant Contact with an Unknown Surface

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Abstract—This work deals with the problem of force regulation and position trajectory tracking for a robot in compliant contact under kinematic uncertainties. A robotic finger with a soft hemispherical tip of uncertain compliance is considered in contact with a rigid flat surface of unknown position and orientation. A novel adaptive controller is proposed and is proved to achieve the convergence of force and position errors to zero by identifying the slope given a persistently excited desired velocity. The performance of the proposed controller is demonstrated by a simulation example.

I. INTRODUCTION

Many applications of robots involve tasks in which the robot end effector is in contact with the environment. In such tasks, the end-effector position and the interaction force between the robot and the environment, have to be simultaneously controlled to achieve either setpoint targets, or desired trajectories. In the force and motion control problems, kinematic uncertainties affecting the robot and the contact kinematics or surface Jacobian may appear whenever the shape of the contacted surface and its position are unknown. For a compliant contact, there are further uncertainties regarding the stiffness parameter and friction that can not be ignored in this case.

The majority of the works dealing with surface kinematic uncertainties consider rigid contact between the end effector and the environment [1]–[5]. In the rigid contact case, the end-effector can not move along the surface normal; hence, end-effector velocities lie on the plane tangent to the surface at the contact point. This fact allows the use of tip velocities in the identification of the constraint surface [3]. For a frictionless point contact, the contact force lies on the surface normal and can be directly used to determine the constraint Jacobian [2], [4]. In the more practical case of contact with friction both measurements of velocities and force are used to calculate the surface normal direction [6]. In order to avoid calculation errors and the use of force derivatives in the control law, an adaptive force-motion controller with estimates of the constraint Jacobian is proposed in [5] exploiting the geometrical relationship between force and end-effector velocity. Regarding uncertainties on surface position, vision systems have been additionally used to identify the desired position [3], [4].

In the case of compliant contact, the problem of force/position regulation has been successfully solved following the concepts of hybrid or parallel control, under both dynamic and kinematic uncertainties, concerning the robot model, the stiffness parameter and the surface orientation [7]–[10]. Force and position tracking has also been successfully solved for stiffness parameter uncertainties, [11]–[14] while [15] deals with both stiffness and surface kinematic uncertainties but ignores friction that is usually present during the contact motion.

This work takes into account surface kinematic, stiffness and friction uncertainties and proposes a novel adaptive controller that achieves force regulation, position tracking and surface slope identification in the presence of friction. Simulation results illustrate the theoretical findings.

II. PROBLEM DESCRIPTION

Consider a robot finger with soft hemispherical fingertip of radius \( r \) in contact with a rigid flat surface. Let \( q \in \mathbb{R}^{n_s} \) be the vector of the generalized joint variables and \( \{B\} \) be inertia frame attached at the finger base (Fig. 1). Let the surface frame \( \{s\} \) be attached at some point on the surface; its position is denoted by \( p_s \) and its orientation by matrix \( R_s = \begin{bmatrix} n_s & o_s & \alpha_s \end{bmatrix} \) such that \( n_s \in \mathbb{R}^3 \) is the unit vector normal to the contact surface pointing inwards. Consider also the frame \( \{t\} \) at the finger rigid tip with position \( p_t \in \mathbb{R}^3 \) and rotation matrix \( R_t \) that can be parameterized by the vector of the rotation angles \( \varphi_t \in \mathbb{R}^3 \) around the axis of the inertia frame. Let the generalized rigid tip position be \( p = \begin{bmatrix} p_t^T & \varphi_t^T \end{bmatrix}^T \in \mathbb{R}^6 \). The generalized velocity \( \dot{p} = \begin{bmatrix} \dot{p}_t^T & \dot{\varphi}_t^T \end{bmatrix}^T \in \mathbb{R}^6 \) is related to the joint velocity \( \dot{q} \) through the rigid tip Jacobian

\[
J = \begin{bmatrix} J_v^T & J_\omega^T \end{bmatrix} \in \mathbb{R}^{6\times n_s}
\]

as follows:

\[
\dot{p} = J(q)\dot{q}
\] (1)

At the center point of the contact area, we attach the contact frame \( \{c\} \), described by the position vector \( p_c \) and
the orientation matrix \( \mathbf{R}_{c} = \mathbf{R}_{s} \). For a soft hemispherical fingertip, the contact point identifies with the rigid tip projection on the surface i.e., \( Q_s \mathbf{p}_c = Q_s \mathbf{p}_t \) where \( Q_s = I - n_s n_s^T \) is the projection matrix. Moreover, for a flat surface the projections of \( \mathbf{p}_c \) and \( \mathbf{p}_s \) along the surface normal are equal \( (n_s^T \mathbf{p}_c = n_s^T \mathbf{p}_s) \) and hence we can derive the material deformation as a function of \( \mathbf{p}_s \), \( \mathbf{p}_c \):

\[
\Delta \mathbf{x} = \mathbf{r} - n_s^T (\mathbf{p}_s - \mathbf{p}_t)
\]

(2)

Fig. 1. A robot finger with a soft hemispherical fingertip in contact with a rigid surface.

We reasonably assume that the integral of the stresses that are distributed nearly semi-circularly over the contact area of the deformed fingertip arises as an aggregated force perpendicular to the surface at the contact point. The concentrated force can be expressed as \( F_c = n_s f \), where the force magnitude \( f \) is assumed to be measurable by a sensor attached at the fingertip [16] and is in general a monotonically increasing, continuously differentiable, nonlinear function of \( \Delta \mathbf{x} : f(\Delta \mathbf{x}) \) if \( \Delta \mathbf{x} > 0, f = 0 \) if \( \Delta \mathbf{x} \leq 0 \). A typical force-deformation relationship is

\[
f(\Delta \mathbf{x}) = k_s \Delta \mathbf{x}, \quad \mu \in \mathbb{R}^+, \quad k_s \text{ is the elasticity model constant.}
\]

Without loss of generality we consider the case of a linear elasticity model that implies \( f = k_s \Delta \mathbf{x}, \quad f = k_s \Delta \mathbf{x} \). The contact force \( F_c \) is mapped to the rigid tip as a generalized force \( F = n f \in \mathbb{R}^d \), with \( n = [n_s^T \mathbf{0}_3^T]^T \in \mathbb{R}^9 \) denoting the generalized normal direction.

In this work, we consider the force regulation and position trajectory following problem under uncertainties arising from surface position and orientation. We further assume that although the force deformation relationship takes the typical structure given above the elasticity model constant may be unknown. Uncertainties on the surface position and orientation affect the accurate definition of the desired position trajectory \( \mathbf{p}_{cd}(t) \) in the three dimensional space. Furthermore, measurements of the contact point position can not be easily obtained in the case of an area contact. These problems can be easily overcome if the surface orientation is known. Then, a trajectory defined in the three-dimensional space can be projected on the surface \( Q_s \mathbf{p}_{cd} \) to annihilate errors that may exist along the surface normal direction; the error \( Q_s(\mathbf{p}_c - \mathbf{p}_{cd}) \) can be used for control purposes as the rigid tip projection on the surface identifies with the contact point i.e. \( Q_s \mathbf{p}_c = Q_s \mathbf{p}_t \). Since the surface orientation is uncertain in this work, the basic idea is to design a controller to achieve both slope identification and force/position targets. In fact, the controller is designed using online estimates of the unknown parameters of surface orientation \( n_s \) (\( Q_s \)) and achieves desired constant force \( f_d \in \mathbb{R}^d \) and position tracking \( \mathbf{p}_d(t) = [p_{cd}^T(t) \quad \omega_{cd}^T(t)]^T \in \mathbb{R}^9 \) by ensuring the convergence of \( n_s \) to its actual value.

We assume that joint positions and velocities are measured and that the rigid tip Jacobian \( J \) is known; hence, rigid tip position and velocity can be calculated using the robot forward kinematic relationships. We further assume that the robot dynamic model is known. The dynamic model of the robot can be written as follows:

\[
M(q) \ddot{q} + \left( \frac{1}{2} \dot{M}(q) + S(q, \dot{q}) \right) \dot{q} + g(q) + J^T C_p \dot{q} + J^T f_n = u
\]

(3)

where \( M(q) \in \mathbb{R}^{n_s \times n_s} \) is the robot inertia matrix that is positive definite, \( S(q, \dot{q}) \in \mathbb{R}^{n_s \times n_s} \) is a skew symmetric matrix, \( C = \dot{q} \text{diag}[c_i] \dot{q} \) is the viscous friction matrix with \( Q = I_6 - mn^T, c_i \) the friction coefficients, \( g(q) \in \mathbb{R}^{n_s} \) denotes the gravity vector and \( u \) denotes the input control law. We assume that friction forces can be linearly expressed with respect to a set of parameters \( \theta_c \in \mathbb{R}^d \) as follows:

\[
J^T C_p = Z_c(q, \dot{q}) \theta_c
\]

(4)

where \( Z_c(q, \dot{q}) \in \mathbb{R}^{n_s \times j} \) is the regression matrix.

III. CONTROLLER DESIGN

The generalized estimate of normal direction denoted by \( \tilde{n} \) is defined according to the definition of \( n \) i.e. \( \tilde{n} = [n_s^T \mathbf{0}_3^T]^T \). The Euclidean norm of \( \tilde{n} \) denoted by \( ||\tilde{n}|| \) depends on the adaptation law and can take non-unit values. However, if the adaptation law is such that \( ||\tilde{n}|| \neq 0 \), the estimate of the generalized projection matrix \( Q \) can be defined as

\[
\tilde{Q} = I_6 - \frac{\tilde{n} \tilde{n}^T}{||\tilde{n}||^2}
\]

(5)

to correspond to a real projection matrix that enjoys the basic properties of symmetry and idempotence. Note that \( \tilde{Q} \tilde{n} = 0 \).

Let us define the reference velocity vector \( \tilde{\mathbf{p}}_r \in \mathbb{R}^9 \) in the rigid tip operational space:

\[
\tilde{\mathbf{p}}_r = \tilde{Q} (\dot{p}_d - \alpha \Delta \mathbf{p}) - \beta \frac{\tilde{n}}{||\tilde{n}||} \Delta \mathbf{f}
\]

(6)

where \( \alpha, \beta \) are positive control gains, \( \Delta \mathbf{p} = p - p_d \) is the position error and \( \Delta \mathbf{f} = f - \tilde{f}_d \) is the force error. The reference acceleration \( \tilde{\mathbf{p}}_r \) is found by taking the derivative of (6):

\[
\tilde{\mathbf{p}}_r = \frac{d}{dt} \left[ \tilde{Q} (\dot{p}_d - \alpha \Delta \mathbf{p}) + \dot{\tilde{Q}} (\ddot{p}_d - \alpha \Delta \ddot{p}) \right] - \beta \frac{d}{dt} \left[ \frac{\tilde{n}}{||\tilde{n}||} \right] \Delta \mathbf{f} - \beta \frac{\tilde{n}}{||\tilde{n}||} \ddot{f}
\]

(7)
where \( \frac{d}{dt} [\cdot] \) denotes the time derivative of a matrix, vector or scalar. The derivatives of the quantities appearing in (7) are calculated as follows:

\[
\frac{d}{dt} \left[ Q \right] = \frac{1}{||n||^2} \left[ 2n^T \dot{n} \dot{m}^T - ||n||^2 \left( \dot{m}^T + \ddot{m}^T \right) \right]
\]
\[
\frac{d}{dt} \left[ \frac{n}{||n||^2} \right] = \frac{1}{||n||^2} \left[ \ddot{n} - 2n^T \dot{n} \right]
\]

(8)

where \( \dot{n} \triangleq \left[ \dot{n}_T \quad 0 \right]^T \) are given by update laws that will be defined through the subsequent stability analysis. The reference acceleration (7) cannot however be used in the control law because the force derivative cannot be accurately obtained. Instead, the reference acceleration can be calculated using an estimate of the force derivative that can be produced given the known structure of the elasticity model and an estimate of the stiffness vector \( \theta_s \) denoted by \( \tilde{\theta}_s \). In the case of the linear force-deformation model \( \theta_s = k_s n_s \). In fact, using the time derivative of (2), the estimate of force derivative is defined as follows:

\[
\dot{\tilde{f}} = \tilde{p}_f^T \tilde{\theta}_s
\]

Using (9), we can define an estimate of the reference acceleration:

\[
\tilde{\tilde{\nu}_r} = \frac{d}{dt} \left[ Q \right] (\dot{\tilde{\nu}}_d - \alpha \Delta p) + Q(\dot{\tilde{\nu}}_d - \alpha \Delta \dot{p}) - \beta \frac{d}{dt} \left[ \frac{n}{||n||^2} \right] \Delta f - \beta \frac{\dot{n}}{||n||^2} \tilde{p}_f^T \theta_s
\]

(10)

that can be expressed as follows:

\[
\tilde{\tilde{\nu}_r} = \tilde{\tilde{\nu}_r} + \beta \frac{\dot{n}}{||n||^2} \tilde{p}_f^T \theta_s
\]

(11)

where \( \tilde{\theta}_s = \theta_s - \tilde{\theta}_s \) is the parameter error.

We also define the error:

\[
s_p = \tilde{\nu} - \tilde{\tilde{\nu}_r}
\]

(12)

and using

\[
\dot{\tilde{\nu}} = \dot{Q} \dot{\tilde{\nu}} + \frac{\dot{n}}{||n||^2} n^T \dot{p} - \frac{\dot{n}}{||n||^2} \dot{\tilde{\nu}}^T \dot{p}
\]

(13)

where \( \tilde{n} = [\tilde{n}_T \quad 0]^T \) with \( \tilde{n}_s = n_s - \tilde{n}_s \), we can express \( s_p \) as follows:

\[
s_p = \tilde{s}_Q + \tilde{s}_n
\]

(14)

where

\[
\tilde{s}_Q = Q(\Delta \dot{p} + \alpha \Delta p)
\]

(15)

\[
\tilde{s}_n = \frac{\dot{n}}{||n||^2} (\Delta \dot{\tilde{\nu}} + \beta \Delta f - \dot{p}_f^T \tilde{n})
\]

(16)

On the other hand \( s_p \) can also be expressed as follows:

\[
s_p = \tilde{p} + \alpha \dot{Q} \dot{p} + v(\tilde{n}, \tilde{\nu}_d, \tilde{p}_d, \Delta f)
\]

(17)

where

\[
v = \frac{\dot{n}}{||n||^2} \beta \Delta f - Q(\dot{\tilde{\nu}}_d + \alpha \Delta p)
\]

(18)

In the non-redundant case, we can also define the reference joint velocity \( \dot{\tilde{\nu}}_r \) as:

\[
\dot{\tilde{\nu}}_r = J^{-1} \tilde{p}_r
\]

(19)

and the error \( s \) as follows:

\[
s = \dot{\tilde{\nu}} - \dot{\tilde{\nu}}_r = J^{-1} s_p
\]

(20)

Using (7) and (19), the reference joint acceleration vector is given by:

\[
\ddot{\tilde{\nu}}_r = -J^{-1} \dot{\tilde{\nu}}_r + J^{-1} \ddot{p}_r
\]

(21)

and its estimate is given by:

\[
\ddot{\tilde{\nu}}_r = -J^{-1} \dot{\tilde{\nu}}_r + J^{-1} \ddot{p}_r
\]

(22)

and can be expressed as follows:

\[
\ddot{\tilde{\nu}}_r = \ddot{\tilde{\nu}}_r + \beta J^{-1} \frac{\dot{n}}{||n||^2} \tilde{p}_f^T \tilde{\theta}_s
\]

(23)

Now, we can propose the following model based input control law:

\[
u = J^T \tilde{n}(\tilde{f}_d - k_f \Delta f) - D_s + M(q) \tilde{\dot{\nu}}_r + \left( \frac{1}{2} \tilde{M}(q) + \tilde{S}(q, \tilde{\nu}) \right) \tilde{\nu}_r + \zeta_c (q, \tilde{\nu}_r) \tilde{\theta}_c + g
\]

(24)

The controller consists of a feedforward term of the desired force magnitude and a proportional term of \( \Delta f \) that lie on the estimated normal direction as well as a feedback of \( s \) through the positive definite gain matrix \( D \). Substituting the input control law (24) into the robot dynamic model (3) we obtain the closed loop system:

\[
M \tilde{s} + \left( \frac{1}{2} \tilde{M} + \tilde{S} \right) s + J^T C s_p + J^T \tilde{n} k_f \Delta f D_s + Z_c(q, \tilde{\nu}_r) \tilde{\theta}_c + J \tilde{n} \tilde{\theta}_s - J M \tilde{\theta}_s = 0
\]

(25)

where \( k_f = k_f + 1 \). Taking the inner product of the closed loop system (25) with \( s \) we get:

\[
\frac{d}{dt} \left[ \frac{1}{2} s^T M s + \frac{1}{2} k_f I (\tilde{\delta} x) \right] + s^T Z_c \tilde{\theta}_c
\]

\[
+ (J s_p - k_f J f_p)^T \tilde{n} - \beta \tilde{s}_T M J^{-1} \frac{\dot{n}}{||n||^2} \tilde{p}_f^T \tilde{\theta}_s
\]

\[
+ \beta k_f \Delta f^2 + s^T D_s + s^T C s_p = 0
\]

(26)

where \( I (\tilde{\delta} x) = \int_0^{\delta x} \{ f(\xi + \Delta x_d) - f_d \} d\xi \) is the potential owing to deformation error with \( I (\tilde{\delta} x) > 0 \) for \( \tilde{\delta} x \neq 0 \). For the case of linear force-deformation relationship the potential is given by \( I (\tilde{\delta} x) = \frac{1}{2} k_c \tilde{\delta} x^2 \). The parameter update laws are chosen as follows:

\[
\dot{\tilde{\theta}}_c = -\Gamma_c Z_c^T \tilde{s}
\]

(27)

\[
\dot{\tilde{s}}_s = -P \left( \Gamma_n \left[ J_3 \quad O_{3 \times 3} \right] \left( f s_p - k_f J f_p \right) \right)
\]

(28)

\[
\dot{\tilde{\theta}}_s = \Gamma_s \tilde{p}_f \frac{\dot{n}}{||n||^2} J^T M s
\]

(29)

where \( \Gamma_c = \text{diag} \left[ \gamma_c^{i-1} \right] \), \( \Gamma_n = \text{diag} \left[ \gamma_n^{i-1} \right] \), \( \Gamma_s = \text{diag} \left[ \gamma_s^{i-1} \right] \) are diagonal matrices of parameter update gains and \( P \) is an appropriately designed projection operator [17] with respect to a convex set \( \tilde{S} \). An example of such a set is defined below:

\[
\tilde{S} = \{ \tilde{n}_s \in \mathbb{R}^3, \quad \tilde{n}_s^T (0) \tilde{n}_s > 0, \quad \tilde{n}_s^T P \tilde{n}_s \geq \varepsilon \}
\]

(30)
where $P = \hat{R}_n(0)\text{diag}[1, -1, -1]\hat{R}_n(0)$ with $\hat{R}_n(0)$ is the initial estimate of the surface rotation matrix and $\varepsilon$ is a positive constant. In fact, this convex set is a rectangular hyperbola on the $n_t(0)$ direction. The actual unit normal vector belong in this set, provided that the absolute value of the angle formed between the initial estimate and the actual normal vector defined by $\phi(t) = a\arccos(n_{\text{norm}})$ is initially less than $45^\circ$ and the constant $\varepsilon$ is chosen arbitrarily small (Fig. 2). The projection operator ensures that $\|\hat{n}_t\| \neq 0$ and $|\phi(t)| < 90^\circ$.

Using (27)-(29) in (26) we get:

$$\frac{dV}{dt} + W = 0 \quad (31)$$

where

$$V = \frac{1}{2}s^TMs + k_f'\Delta f + \frac{1}{2}\hat{\theta}_cT\hat{\theta}_c$$

$$+ \frac{1}{2}n_T^T\hat{n}_s + \frac{1}{2}n_T^T\hat{n}_s$$

$$W = \beta k_f'\Delta f + s^T D_s + s^T C_s p$$

Function $V$ is positive definite with respect to $s$, $\Delta f$ and parameter errors $\hat{\theta}_c$, $\hat{\theta}_s$ and $\hat{n}_s$ while function $W$ is positive definite with respect to $s$, $\Delta f$. Hence, function $V$ has a negative derivative i.e. $\frac{dV}{dt} = -W \leq 0$ and can be regarded as a Lyapunov-like candidate function in order to prove the following theorem.

**Theorem:** The input control law (24) with the update laws (27)-(29) applied in (3) achieves boundedness of all signals and error convergence to zero as well as slope identification given the persistent excitation of $p_d$ ($\Delta f \to 0$, $\Delta x \to 0$, $Q\Delta p \to 0$, $\Delta \dot{p} \to 0$ and $\hat{n}_s \to 0$).

**Proof:** $\frac{dV}{dt} = -W \leq 0$ implies $V(t) \leq V(0)$ and hence $s$ ($p_d$, $\hat{s}_Q$, $\hat{s}_n$), $\Delta x$ ($\Delta f$), $\hat{\theta}_c$, $\hat{\theta}_s$ and $\hat{n}_s$ ($\hat{n}$) in $\mathcal{L}_\infty$. Hence, given $p_d$, $\hat{p}_d$ are bounded trajectories, $v \in \mathcal{L}_\infty$ and consequently (17) implies $\dot{p} + \alpha Q \hat{p} \in \mathcal{L}_\infty$. Since $\hat{Q}$ is bounded and positive semi-definite, it can be easily proved that $p, \hat{p} \in \mathcal{L}_\infty$ and in turn $\Delta \hat{x} \in \mathcal{L}_\infty$. If the robot moves away from singular positions, the boundedness of $\hat{p}$ implies that $\hat{q}$ is bounded. From (25), $\hat{s}$ can be expressed as a sum of bounded quantities and hence $s \in \mathcal{L}_\infty$. From (31) and (33) $\Delta f$, $s$ and their derivatives implies that $\Delta f$, $s$ are uniformly continuous and it follows from Desoer and Vidyasagar (1975) that $\Delta f \to 0$ ($\Delta x \to 0$), $s \to 0$ ($\hat{s}_p$, $s_t$, $\hat{s}_Q \to 0$). From (17), the boundedness of $\hat{s}_p$ implies that $\hat{p}$ is bounded as a function of bounded quantities and hence $\hat{p}$ ($\Delta \hat{x}$) is uniformly continuous. Furthermore, (25) implies that $\hat{s}$ is uniformly continuous as a function of uniformly continuous quantities. The uniform continuity of $\Delta \hat{x}$, $\hat{s}$ as well as the convergence of $\dot{\Delta} x$, $\hat{s}$ to zero implies $\Delta \dot{\hat{x}}$, $\hat{s} \to 0$. Furthermore, using (27)-(29) the convergence of $s$ and $\Delta f$ implies that $\hat{\theta}_c$, $\hat{\theta}_s$, $\hat{n}_s \to 0$ and given (8) we get $\hat{Q} \to 0$ and in turn $\hat{s}_Q \to \frac{\alpha}{\delta}[\hat{Q}\Delta \hat{p}] + \alpha Q \Delta \hat{p} \to 0$; hence, $Q\Delta \hat{p}, \Delta \hat{p} \to 0$.

Using $\hat{p}_d \to \hat{p}_d$ and $\Delta \hat{x} = n^nT \hat{n} \to 0$ the rigid tip velocity $\hat{p}$ converges to $(I - \cos()n^nT)\hat{p}_d$ since $\cos$ can not take zero values owing to the use of the projection operator in the update law (28). Consequently, as $\hat{s}_n$ given by (16) converges to zero the quantity $\hat{p}_d \hat{p}_d(n - \hat{n}n^n)$ converges to zero and $\hat{n} \to \hat{n}_0$ provided that $p_d$ is a persistently excited signal, i.e.

$$\int_{t}^{t+T_0} \hat{p}_d \hat{p}_d (\tau) \hat{p}_d (\tau) d\tau \geq \epsilon \alpha T_0 I, \quad \forall t \geq 0$$

for some $\epsilon > 0$, $T_0 > 0$. The convergence of $\hat{n}$ to its actual value implies that $Q \to Q$ and in turn $Q\Delta \hat{p}, \Delta \hat{p} \to 0$. Notice that the convergence of $\hat{\theta}_c$, $\hat{\theta}_s$ is not required.

**Remark 1:** The theorem holds even when $k_f = 0$. However, the use of a proportional term for the force error in (24) provides an extra degree of freedom to the designer for improving the performance of the system response through the appropriate tuning of $k_f$.

**Remark 2:** For the more general case of a non-linear force-relationship deformation $f = k_s \Delta x$, the parameter vector includes $k_s$, $n_s$ and the surface position $n^n_T p_d$.

**Fig. 3:** Contact maintenance and potential owing to deformation error

Notice that the theorem is valid provided that the contact between the robotic finger and the surface is maintained i.e. $\Delta x > -\Delta x_d$, $\forall t \geq 0$. This corresponds to an upper bound for the potential owing to deformation error, i.e. $I(\Delta x) < f_d \Delta x_d - \int_0^{\Delta x} f(\xi) d\xi$, $\forall t \geq 0$ (Fig. 3) and in turn $V < \int f_d \Delta x_d - \int_0^{\Delta x} f(\xi) d\xi$, $\forall t \geq 0$. Since $V$ is a decreasing function i.e. $V(0) \geq V(t), \forall t \geq 0$, starting within the region $V(0) < \int f_d \Delta x_d - \int_0^{\Delta x} f(\xi) d\xi$, $\forall t \geq 0$. For the case of linear force-deformation relationship the condition can be written as: $V(0) < \frac{1}{2}k_f f_d \Delta x_d$ and ensures both contact maintenance and less than 100% force overshoot.
IV. SIMULATION RESULTS

Consider a two-dof planar manipulator with link lengths $l_1 = 0.3(m)$, $l_2 = 0.2(m)$, masses $m_1 = 0.8(kg)$, $m_2 = 0.6(kg)$ and inertias $I_1 = \frac{m_1 l_1^2}{12}$, $I_2 = \frac{m_2 l_2^2}{12}$. The surface is modeled by a line with slope $\varphi_s = 60^\circ$ with a normal vector $\mathbf{n}_s = [\sin \varphi_s \quad -\cos \varphi_s]^T$. Normal force magnitude is simulated by $f = k_s \Delta x$, where $k_s = 10,000$ while friction coefficients $c_i$ are equal to 5. The end-effector initial contact point position is $p_i(0) = \begin{bmatrix} 0.3575 \\ 0.14 \end{bmatrix}^T (m)$ and exerts a normal force of $f(0) = 4 N$. The control purpose is to exert a normal force with magnitude $f_d = 9 N$ and to track the desired position trajectory $p_d(t) = P_d + A_d \sin(t)$, where $P_d = \begin{bmatrix} 0.3575 \\ 0.12 \end{bmatrix}^T (m)$ and $P_d = \begin{bmatrix} 0.02 \\ 0.03 \end{bmatrix}^T (m)$ that corresponds to an initial error of $[\varphi_s^T \Delta p] = 0.0173(m)$ without loosing contact. The initial estimate of the line slope given in

the adaptive law is $\hat{\varphi}_s(0) = 50^\circ$ that corresponds to an initial angle error $10^\circ$. The initial estimates of elasticity model constant and friction coefficients are $k_s = 8,000$ and $c_i = 4$ respectively, that are 20% smaller than the actual values. The friction parameter vector is $\mathbf{\theta}_f = [c_1 \cos \varphi_s \\ c_2 \sin \varphi_s \\ c_1 \cos \varphi_s \sin \varphi_s]^T$ while the friction regression matrix is:

$$
\mathbf{Z}_f^T = \begin{bmatrix}
-1 \qquad 1 \\
1 \qquad 1 \\
1 \qquad 1
\end{bmatrix}
$$

where $s_1 = \sin q_1$, $s_1 = \sin(q_1 + q_2)$, $c_1 = \cos q_1$, $c_2 = \cos(q_1 + q_2)$. The estimates $c$, $k_s$, $\varphi_s$ are used to calculate $\theta_f(0), \theta_f(0), \mathbf{n}_s(0)$. The gains of the controller have the following values: $\alpha = 60, \beta = 0.8, k_f = 0$, $D = J^T \text{diag}[0.007, 0.005, 0.01, 0.007, 0.007]$, $\Gamma_c = \text{diag}[0.05, 0.5, 0.1, 0.00003, 0.0005]$, $\Gamma_n = \text{diag}[0.05, 0.5, 0.1]$, $\Gamma_s = \text{diag}[0.0003, 0.0005]$

Fig. 4 shows the desired and actual tangent position trajectories. Fig. 5 and 6 show the convergence of force error to zero and the estimated normal vector coordinates to their actual values respectively. Within the first period the position tracking error and the force error are reduced to the order of $10^{-4}$; in the second period they have reached the order of $10^{-5}$. Notice that the surface slope has been satisfactorily identified within the first period where the main transients of position and force errors lie. On the other hand, stiffness parameters $\hat{\theta}_s$ do not converge to their actual values as shown in Fig. 7.

V. CONCLUSIONS

This work proposes an adaptive controller for the problem of force regulation and position tracking to cope with parametric uncertainties in surface kinematics, fingertip compliance and dynamic friction. The convergence of force and position errors to zero is achieved together with the identification of the surface slope provided that the desired velocity trajectory is a persistently excited signal.
The control scheme requires measurements of joint variables and force magnitude. The proposed approach can be easily extended to include dynamic model uncertainties. Future work will extend this approach to the case of both force and position tracking.

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